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Lottery versus share contests under risk aversion☆

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Abstract

Lottery and share contests are equivalent for risk neutral contestants. We compare these two contests designs to show that this equivalence does no longer hold for risk averse contestants, in a policy contest setting. As expected, they prefer the share contest as it eliminates the uncertainty of the lottery. Under institutional settings in which contestants can pre-commit to policies different from their ideal one, the previous result is switched: Risk-averse contestants prefer lottery contests because, only under this design, they strategically moderate their claims, calming down the conflict and reducing the uncertainty of the lottery. Moreover, we show that contestants exert more effort in share contests. These results provide arguments justifying each of these two types of contests depending on the institutional framework and the comparative criteria.

Keywords: lobbying; lottery contest; share contest; risk aversion; commitment; strategic restraint.
JEL Classification: C72 D72 D81 H40

1. Introduction

In a wide variety of economic situations, contestants compete for a prize. Sometimes this prize is indivisible; for example, agents disputing a government contract, firms aiming to become the monopolist in a market, workers competing for a job position, or students applying for college admissions. In other cases, the prize can be shared among contenders; e.g. emission quotas that should be allocated among the firms of an industry, research funds among universities, or tax revenues among either ethnic/social groups, economic sectors, or regions of a country. In conflicts between lobbyists who seek to influence a regulator's policy choice, we can establish an analogous classification of confrontations. In some of them, the regulator

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has to make a dichotomous decision, so that each lobbyist can either get the whole stake or not, as in an indivisible-prize contest. Public authorities must make these decisions sometimes. For example, when a public agency decides whether to accept the demands of a claimant or a public administration chooses between building a new facility or not. In other cases, the policy choice can take a continuum of possible values—as in Dickson et al. (2018) or Duggan & Gao (2019). For example, in an environmental dispute, activists and industrialists aim to influence the legal level of emissions ranging over a continuum. The regulator's decision is of the same nature in a conflict among interest groups about the location of a public facility, about immigration quotas, or about the tax burden on a specific economic activity or income group. Even disputes with apparently two resolutions may finally have an intermediate verdict: In the dispute about whether slaves would be counted for legislative representation and taxing purposes, state delegates reached the Three-Fifths Compromise in 1787, meaning that five slaves were counted as three non-slaves. In all these cases, as in a divisible-prize contest, the stake can be shared among participants because the regulator's policy choice can take any intermediate position between the preferred policies of the lobbyists. In this type of situations, where the policy potentially ranges over a continuum, the contest can be framed into two distinct designs: either a lottery contest, in which the regulator will select probabilistically the policy supported by one lobbyist, or a share contest, where the policy outcome will be a weighted combination of lobbyists' claims. The specific design of the contest can be determined by tradition, law or political culture but also, as pointed out by Che & Gale (1997), it can be chosen by a contest designer (either a politician or a bureaucrat), as she may restrict her choice to a dichotomy or may let room for intermediate resolutions. We can even think lobbyists' may exert some influence on the choice of this contest designer. Our paper provides rationales for these two alternative types of lobbying contests, justifying their appropriateness depending on the institutional setting and the specific criteria used to compare them.

Regarding the institutional circumstances that affect the comparison between these two contest designs, we analyze two alternative scenarios: In one of them, lobbyists cannot credibly commit to support a policy different from their ideal one and in the alternative they can. These two settings can reflect two different levels of lobbyists' reputation, credibility or, they can be interpreted as two distinctive environments: one in which lobbying processes are not institutionalized or institutions are weak and another where these institutions are strong enough to guarantee compliance of the lobbyists' commitments. Referring the specific criteria under which we can compare lottery and share contests, there are several options to consider. Many theoretical papers compare alternative contests in terms of rent dissipation or contestants' total effort (e.g. Gradstein & Konrad, 1999; Baik & Lee, 2000; Fu & Lu, 2007). In some lobbying processes, the regulator can receive (part of) these efforts as bribes. Thus, he or she might be interested in maximizing this aggregate effort. In others, contestants' efforts are sunk costs, so these efforts reduce aggregate welfare. In light of this alternative criteria, we compare lottery and share contests in terms of contestants' individual and aggregate efforts and, according to their equilibria utilities.

Lobbyists are indifferent between lottery and share contests when they are risk neutral. In this study, we analyze these two designs in a policy contest framework with two risk-averse lobbyists. We assume that their marginal cost of effort is constant and might coincide across lobbyists (symmetric contest) or not (asymmetric contest). As expected, risk aversion breaks the above equivalence between the two contest designs: When lobbyists cannot credibly commit to a policy different from their ideal one, the share contest is welfare enhancing with respect to the lottery contest as it eliminates the uncertainty of the probabilistic policy choice of the regulator. The fear of risk-averse lobbyists to this uncertainty pushes them to reduce the stake of the lottery contest when they have the opportunity to do so; i.e. when they can credibly commit to support policies other than their ideal one. They strategically shrink the stake by claiming policies that are closer to the opponent's position. In other words, lobbyists moderate their policy claims. As showed by Epstein & Nitzan (2004), lobbyists prefer to make this concession because it lowers the level of conflict and increases their own winning probability. This key fact reverses the welfare comparison between lottery and share contest designs with respect to the previous institutional framework: The possibility of committing to certain policy proposals involves that lobbyists are better off under the lottery contest design because the strategic moderation of claims mitigates the extra uncertainty of the lottery, cools down the confrontation and alleviates the effort cost burden of the contest. The institutional frame turns out to be crucial to determine the best contest design according to lobbyists' welfare. In terms of aggregate lobbying efforts, we show that the share contest involves a greater aggregate effort (aggregate effort levels coincide in symmetric contests when lobbyists cannot pre-commit). This is driven by the larger effort of the disadvantaged contestant—i.e. the one facing a higher marginal cost of effort—who is encouraged by the share-contest design.

We relate our comparison between these two alternative types of contests to the experimental literature on rent-seeking contests (e.g. Chowdhury et al., 2004 or Cason et al., 2018, among others). A common finding in all these papers is that the observed aggregate effort in lottery contests exceeds the theoretical equilibrium effort for risk neutral agents. This overbidding is reduced in share contests. Chowdhury et al. (2004) point out several reasons that could explain the lower effort in the share contest: (i) the utility of winning can be lower in the share contest as there is not a clear winner, (ii) proportional prizes may enhance learning incentives, and (iii) behavior could be affected by the lower risk of the share contest. To deal with the effects of risk on the equilibrium behavior, they will appear as long as agents are not risk neutral. However, to the best of our knowledge, the comparative analysis between share and lottery contests for non-risk-neutral agents remains unexplored in the theoretical literature. Our analysis aims to fill this gap by comparing these two types of contests assuming risk averse agents.

A related aspect that has been analyzed by the literature concerns the effects of risk aversion on equilibrium behavior. Several empirical papers have shown a (negative) relationship between risk aversion and effort in contests (Millner & Pratt, 1991; Anderson & Freeborn, 2010; Sheremeta & Zhang, 2010; Sheremeta, 2011; Cason et al., 2018). From a theoretical viewpoint, the direction of these effects is ambiguous (Hillman & Katz,1984; Van Long & Vousden, 1987; Skaperdas & Gan, 1995; Konrad & Schlesinger, 1997; Cornes & Hartley, 2009; Treich, 2010). Unlike this literature, that compares the equilibrium features (frequently rent-dissipation) among alternative contestants with various risk attitudes, we develop a comparison between different contest designs for the same contestants (i.e. contestants with the same risk preferences). This allows us to add a comparative statics welfare analysis.

The rest of the paper is organized as follows. Section 2 presents the setting. In section 3, we present our results under the two alternative institutional settings. Section 4 concludes.

2. The contests

Consider two lobbyists L and R seeking to influence the regulator's choice of a policy $y \in [0,1]$. These lobbyists simultaneously exert efforts $x_L, x_R \in \mathbb{R}_+$ to support their policy claims $y_L, y_R \in [0,1]$ ($y_L \leq y_R$). They are risk-averse; their preferences over policies are represented by the utility function $u_i(y, x_i) = k_i - (\tilde{y}_i - y)^2 - \alpha_i x_i$, where $i \in \{L, R\}$ and $\tilde{y}_L = 0$ and $\tilde{y}_R = 1$ are the lobbyists' ideal policies. Without loss of generality we assume $\alpha_L = \alpha$, where $\alpha \in (0,1]$, and $\alpha_R = 1$.[1] Then, lobbyist L (potentially) has a cost advantage over R. We consider two alternative institutional frameworks. In one of them, lobbyists cannot credibly commit to support a policy different from their favorite ones, so that $y_L = 0$ and $y_R = 1$ (No-Commitment or NC-case). In the other, they can credibly set policy claims different from $\tilde{y}_L$ and $\tilde{y}_R$ (Commitment or C-case).

These efforts affect the regulator's policy choice via one of the two following contest designs: In one of them, the regulator's policy choice is dichotomous, $y \in \{y_L, y_R\}$. Here, each contestant's effort affects the probability of the regulator choosing her respective claim according to a standard Tullock function. Thus, contestants' expected utilities are:

$$\begin{aligned} U_L^l(x_L, x_R; y_L, y_R) &= p_L u_L(y_L, x_L) + p_R u_L(y_R, x_L) \\ U_R^l(x_R, x_L; y_L, y_R) &= p_L u_R(y_L, x_R) + p_R u_R(y_R, x_R), \end{aligned}$$

[1]Note that this utility representation is general enough to encompass $u_i(y, x_i) = k_i - \beta_i(\tilde{y}_i - y)^2 - \alpha_i x_i$ for any $k_i, \beta_i, \alpha_i \in \mathbb{R}_+$, $i = L, R$.

where $p_L = x_L / (x_L + x_R)$ and $p_R = 1 - p_L$.[2] We can rewrite these utilities as

$$\begin{aligned} U_L^l (x_L, x_R; y_L, y_R) &= p_L D_L (y_L, y_R) + u_L (y_R, x_L) \\ U_R^l (x_R, x_L; y_L, y_R) &= p_R D_R (y_L, y_R) + u_R (y_R, x_R), \end{aligned}$$

where

$$D_L (y_L, y_R) = u_L (y_L, x_L) - u_L (y_R, x_L) = y_R^2 - y_L^2 \text{ and} \tag{1}$$

$$D_R (y_L, y_R) = u_R (y_R, x_R) - u_R (y_L, x_R) = (1 - y_L)^2 - (1 - y_R)^2, \tag{2}$$

are the stakes of the contest. We will refer to this design as a lottery contest.

Under the other contest design, the policy selected by the regulator results from a compromise or a balance of forces between the two lobbyists. Hence, this compromise policy $\overline{y} \in [0, 1]$ is proportional to the lobbying efforts, so that $\overline{y} \equiv p_L y_L + p_R y_R$, where p_L and p_R are as defined above. Thus, contestants' utilities are:

$$\begin{aligned} U_L^s (x_L, x_R; y_L, y_R) &= u_L (\overline{y}, x_L) \\ &= k_L - \left(\frac{x_L}{x_L + x_R} y_L + \frac{x_R}{x_L + x_R} y_R \right)^2 - \alpha x_L \text{ and} \qquad (3) \\ U_R^s (x_R, x_L; y_L, y_R) &= u_R (\overline{y}, x_R) \\ &= k_R - \left(1 - \left(\frac{x_L}{x_L + x_R} y_L + \frac{x_R}{x_L + x_R} y_R \right) \right)^2 - x_R. \qquad (4) \end{aligned}$$

Unlike the previous contest design, in this situation the policy choice is not probabilistic. We will refer to this design as a share contest.

3. Equilibrium analysis

In the lottery contest, the optimal contestant's efforts (x_L^l and x_R^l) must satisfy:

$$\begin{aligned} \frac{\partial U_L^l}{\partial x_L} &= \frac{x_R^l}{\left(x_L^l + x_R^l\right)^2} D_L (y_L, y_R) - \alpha = 0 \text{ and} \\ \frac{\partial U_R^l}{\partial x_R} &= \frac{x_L^l}{\left(x_L^l + x_R^l\right)^2} D_R (y_L, y_R) - 1 = 0. \end{aligned}$$

From these two FOCs, we get that $\alpha x_L^l D_R = x_R^l D_L$,

$$x_R^l = \alpha D_L \left(\frac{D_R}{D_L + \alpha D_R} \right)^2, \; x_L^l = D_R \left(\frac{D_L}{D_L + \alpha D_R} \right)^2, \text{ and } p_L^l = \frac{D_L}{D_L + \alpha D_R}. \tag{5}$$

[2]Note that this is isomorphic to a game in which $u_i (y, x_i) = k_i - (\tilde{y}_i - y)^2 - c_i$ and $p_L = c_L / (c_L + \alpha c_R)$.

This yields,

$$U_L^l\left(x_L; x_R, y_L, y_R\right) = \frac{D_L^3}{\left(\alpha D_R + D_L\right)^2} + k_L - y_R^2 \tag{6}$$

$$U_R^l\left(x_R; x_L, y_L, y_R\right) = \alpha^2 \frac{D_R^3}{\left(\alpha D_R + D_L\right)^2} + k_R - \left(1 - y_L\right)^2. \tag{7}$$

In the share contest, optimal efforts (x_L^s and x_R^s) satisfy

$$\frac{\partial U_L^s}{\partial x_L} = \frac{2\bar{y} x_R^s \left(y_R - y_L\right)}{\left(x_L^s + x_R^s\right)^2} - \alpha = 0 \text{ and} \tag{8}$$

$$\frac{\partial U_R^s}{\partial x_R} = \frac{2\left(1-\bar{y}\right) x_L^s \left(y_R - y_L\right)}{\left(x_L^s + x_R^s\right)^2} - 1 = 0, \tag{9}$$

for $x_L^s, x_R^s > 0$ and $y_R > y_L$. Rearranging these two FOCs, we obtain the following relation between the compromise policy and the equilibrium costs of effort

$$\frac{\bar{y}}{1-\bar{y}} = \frac{\alpha x_L^s}{x_R^s}. \tag{10}$$

Therefore, $\bar{y} = 1/2$ only when the cost of effort of the two contestants coincide.

3.1. No-commitment case

Next we compare the two contest designs when lobbyists' claims are fixed to $y_L = 0$ and $y_R = 1$ in terms of lobbyists' efforts and utilities.

In the lottery contest, $D_L(0,1) = D_R(0,1) = 1$. Therefore:

$$x_L^l = \frac{1}{\left(1+\alpha\right)^2}, \quad x_R^l = \frac{\alpha}{\left(1+\alpha\right)^2}, \quad \text{and} \quad x_L^l + x_R^l = \frac{1}{\left(1+\alpha\right)}.$$

In the symmetric contest ($\alpha = 1$), $x_L^l = x_R^l = 1/4$, yielding equilibrium utilities, $k_i - 3/4$, $i = L, R$. When $\alpha < 1$, $x_L^l > x_R^l$. However, the total cost of effort does not differ across contestants, *i.e.* $\alpha x_L^l = x_R^l$. The equilibrium effort of the advantaged (disadvantaged) contestant L (R) decreases (increases) with α. Aggregate effort decreases with α, *i.e.* the more balanced is the confrontation the higher is the aggregate effort exerted.

In the share contest, $y_L = 0$ and $y_R = 1$ imply that $\bar{y} = x_R / \left(x_L + x_R\right)$. Using this, (10) can be rewritten as $\sqrt{\alpha} x_L^s = x_R^s$. This contrasts with the lottery contest, as the equilibrium cost of effort does no longer coincide across contestants (whenever $\alpha < 1$).

Solving the system of equations (8)-(9), we get that:

$$x_L^s = \frac{2}{\left(1+\sqrt{\alpha}\right)^3}, \quad x_R^s = \frac{2\sqrt{\alpha}}{\left(1+\sqrt{\alpha}\right)^3}, \quad \text{and} \quad x_L^s + x_R^s = \frac{2}{\left(1+\sqrt{\alpha}\right)^2}.$$

The equilibrium compromise policy is $\bar{y} = \sqrt{\alpha}/(1+\sqrt{\alpha})$, going from 1/2 in the symmetric case to zero as α decreases. In the symmetric contest, equilibrium efforts are $x_L^s = x_R^s = 1/4$, as in the lottery contest. However, under asymmetry, individual and aggregate efforts under the share contest will differ from those of the lottery contest. We next summarize our comparisons across contest designs in terms of individual and aggregate efforts.

Lemma 1. *In the NC-case, when $\alpha = 1$, lobbying efforts coincide across both contest designs. For $\alpha \in (0,1)$:*

(a) Aggregate efforts in the share contest are larger than in the lottery contest, i.e. $x_L^s + x_R^s > x_L^l + x_R^l$.

(b) The disadvantaged contestant R exerts more effort in the share contest.

(c) The advantaged contestant L exerts less effort in the share contest if and only if α is sufficiently large ($\alpha > 0.141441$).

Point (a) is illustrated in Figure A.1(b) and says that, for any asymmetric contest, risk averse contestants facing linear costs of effort exert a higher aggregate effort in the share contest than in the lottery contest. Evidence from the experimental literature (Chowdhury et al., 2004; Cason et al., 2018), assuming a convex effort-cost function and symmetric contestants, shows that experimental subjects exert a lower aggregate effort in the share contest. The different functional forms of the cost function might explain (part of) these contrasting results. The other factor that may (partly) clarify the difference between these empirical findings and our theoretical predictions, refers to risk aversion. Although the effects of risk aversion on equilibrium efforts have been studied in the literature, as commented in our introduction, the conclusions regarding the direction of these effects are ambiguous. Thus, the role risk aversion play explaining the difference between our previous result and the experimental findings is still unclear and deserves further attention in future research.

Figure A.1(a) illustrates the contestants' individual effort comparisons of parts (b) and (c) of Lemma 1. Regarding the disadvantaged contestant, it can be seen that the marginal productivity of effort is larger in the share contest than in the lottery contest.[3] Therefore, the share contest encourages the disadvantaged contestant more than the lottery contest. It can be shown that the share contest discourages the advantaged contestant more than the lottery contest. This explains the comparison

[3] For contestant R, the marginal productivity of effort in the share contest is $\frac{\partial U_R^s}{\partial x_R} + 1 = 2(1-\bar{y})(\partial \bar{y}/\partial x_R) = 2(1-\bar{y})(\partial p_R/\partial x_R)$. Her marginal productivity of effort in the lottery contest is $\frac{\partial U_R^l}{\partial x_R} + 1 = D_R \cdot (\partial p_R/\partial x_R) = \partial p_R/\partial x_R$. It is easy to show that $\frac{\partial U_R^s}{\partial x_R} + 1 > \frac{\partial U_R^l}{\partial x_R} + 1$ when $2(1-\bar{y}) > 1$, *i.e.* when $p_L > 1/2$.

between contestants' equilibrium efforts for sufficiently large values of α. We will explore the reasons underlying the shift in this comparison for low values of α after the analysis of contestants' utilities that we present next.

Under the lottery contest design, contestants' probabilities of winning are $p_L^l = 1/(1+\alpha)$ and $p_R^l = \alpha/(1+\alpha)$ and their equilibrium utilities are:

$$\begin{aligned} U_L^l &= k_L - 1 + \frac{1}{(1+\alpha)^2}, \text{ and} \\ U_R^l &= k_R - 1 + \frac{\alpha^2}{(1+\alpha)^2}. \end{aligned}$$

Despite both contestants carry the same cost of effort in equilibrium, the advantaged contestant L has a larger probability of winning and, as a result, enjoys a larger utility for all $\alpha \in (0,1)$. In this lottery contest, we can also compute the expected policy arising from the binary choice of the regulator. This is $\alpha/(1+\alpha)$, going from 1/2 in the symmetric contest to 0, as the asymmetry increases. This shows that as the advantage of L increases, the regulator's probability of choosing y_L approaches to one.

Contestants' equilibrium utilities in the share contest are:

$$U_L^s = k_L - \frac{\alpha(3+\sqrt{\alpha})}{(1+\sqrt{\alpha})^3}, \text{ and} \tag{11}$$

$$U_R^s = k_R - \frac{(1+3\sqrt{\alpha})}{(1+\sqrt{\alpha})^3}. \tag{12}$$

Notice that, for the symmetric case ($\alpha = 1$), apart from the coincidence between individual efforts across contest designs, the expected policy of the lottery contest coincides with the compromise policy $\bar{y}$ of the share contest (both are equal to 1/2). Risk-averse contestants prefer the share contest rather than the lottery contest in this symmetric case, because there is a lower uncertainty. For asymmetric cases, this may no longer hold. Efforts do not coincide and the expected policy of the lottery contest differs from the compromise policy of the share contest, as showed by Figure A.2(a). Therefore, the comparison between contestant's utilities across contest designs will depend on the interplay among (i) the expected/compromise policy, (ii) the uncertainty of the lottery contest, and (iii) the equilibrium effort costs under the two contest designs. The result is summarized by the next Proposition.

Proposition 1. *In the NC-case, when $\alpha = 1$, contestants' equilibrium utilities are larger in the share contest. For $\alpha \in (0,1)$:*

(a) Contestants obtain a larger aggregate utility in the share contest.

(b) The disadvantaged contestant R obtains a larger equilibrium utility in the share contest.

(c) *The advantaged contestant L obtains a higher utility in the share contest unless her advantage is sufficiently large ($\alpha < 0.0356564$).*

Point (a) shows that, under this institutional framework (NC case), the share contest is more appealing in terms of social welfare (sum of the two contestants' utilities). Figure A.3 illustrates parts (b) and (c) of this result. To explain the intuitive reasons underlying this result, we need to consider factors (i), (ii) and (iii).

In terms of the expected/compromise policy (i), the comparison between the two contest designs is unambiguous—see Figure A.2(a): The share contest favors the disadvantaged contestant R (and punishes the advantaged contestant L), especially when the disadvantage is relatively marked (*i.e.* low values of α). Regarding (ii), the probabilistic nature of the lottery contest involves an additional uncertainty compared to the share contest. We can measure this extra risk by the variance of the two policy outcomes in the lottery contest. This variance is $\alpha / (1+\alpha)^2$, going from $1/4$ in the symmetric case to zero as α converges to zero. In respect of factor (iii), Lemma 1 shows that the disadvantaged contestant always exerts more effort in the share contest. Whereas L exerts a lower effort in the share contest, unless her advantage is large enough (α sufficiently small). In relatively even confrontations, the effort exerted by L in the lottery contest is higher than the effort exerted in the share contest to compensate for the extra cost of uncertainty of the lottery. However, when L has an important advantage, L need not make this compensation because the expected policy of the lottery contest is much more favorable for L than the compromise policy of the share contest. As part (b) of Proposition 1 shows, the effect of factors (i) and (ii) exceed that of (iii) for the disadvantaged contestant R. For contestant L, part (c) shows that the effect of factors (ii) and (iii) exceed the effect of (i) when her advantage is not too large (*i.e.* α is not sufficiently small). However, when this advantage is large enough, the effect of (iii) is reversed. Then, the effect of factors (i) and (iii) exceeds that of (ii).

3.2. Commitment case

In this new scenario, lobbyists can credibly set policy claims different from $\tilde{y}_L$ and $\tilde{y}_R$ before the contest stage. Specifically, we consider a two-stage game in which, first, contestants simultaneously decide the policy they will lobby for and, second, they simultaneously choose the effort they will exert to support that policy in a lobbying contest. As in the previous situation, our primary aim is to compare the two contest designs in terms of contestants' efforts and utilities. However, now we first need to analyze the choice of the initial policy claims. In particular, we will explore the contestants' incentives to moderate their claims to lower the intensity of the subsequent conflict. Specifically, contestant i's moderation occurs when i selects to claim a policy that is closer to the opponents' ideal policy than $\tilde{y}_i$. When this happens, we say that i *concedes*.

Let us first consider the lottery contest. Replacing equations (1) and (2) in the expected utility functions of L and R (6)-(7) we get

$$V_L^l(y_L, y_R) = \frac{(y_R - y_L)(y_L + y_R)^3}{(y_L + y_R + \alpha(2 - y_L - y_R))^2} + k_L - y_R^2 \text{ and} \tag{13}$$

$$V_R^l(y_L, y_R) = \frac{\alpha^2 (y_R - y_L)(2 - y_R - y_L)^3}{(y_L + y_R + \alpha(2 - y_L - y_R))^2} + k_R - (1 - y_L)^2. \tag{14}$$

In the symmetric case ($\alpha = 1$), the equilibrium claims satisfy the FOCs

$$\begin{aligned} -(y_L + y_R) + 3(y_R - y_L) &= 0 \text{ and} \\ (2 - y_L - y_R) - 3(y_R - y_L) &= 0, \end{aligned}$$

yielding equilibrium claims $y_L^l = 1/3$ and $y_R^l = 2/3$. Therefore, both contestants make concessions. Using (5), we obtain equilibrium efforts $x_L^l = x_R^l = 1/12$, which are lower than those in the NC-case. Thus, contestants' concessions lead to a cost effort saving. This gains from savings exceed the utility loss of the concession because the equilibrium utilities are higher than those in the NC-case. In this C-case, they are equal to $k_i - 13/36$, $i = L, R$.

For $\alpha \in (0, 1)$, we can rewrite the FOCs as,

$$\begin{aligned} 0 &= (-(y_L + y_R) + 3(y_R - y_L)) M - 2(1 - \alpha)(y_R - y_L)(y_L + y_R) \text{ and} \\ 0 &= 2(1 + y_L - 2y_R)\alpha^2 M (2 - y_L - y_R)^2 \\ &\quad -2\alpha^2 (1 - \alpha)(y_R - y_L)(2 - y_R - y_L)^3, \end{aligned}$$

where $M = (y_L + y_R + \alpha(2 - y_L - y_R)) > 0.$[4]

The equilibrium claims satisfying these FOCs are:

$$y_L^l = \frac{\alpha\left(3(1 + \alpha) - \sqrt{1 + \alpha(34 + \alpha)}\right)}{2(1 - \alpha)^2} \quad \text{and} \quad y_R^l = \frac{\sqrt{1 + \alpha(34 + \alpha)} - 1 - 7\alpha + 2\alpha^2}{2(1 - \alpha)^2},$$

[4] Second-order conditions are satisfied as

$$\frac{\partial^2 V_L^l}{\partial y_L^2} = \frac{-48\alpha^2 y_L (y_L + y_R) - 2(y_L + y_R)^4 (1 - \alpha(2 - \alpha)) - 16\alpha(1 - \alpha)(y_L + y_R)^3}{(y_L + y_R + \alpha(2 - y_L - y_R))^4} < 0$$

and $\partial^2 V_R^l / \partial y_R^2$ evaluated at equilibrium y_L^l and y_R^l equals

$$\frac{\alpha^2\left(\left(9 + 130\alpha + 24\alpha^2 - 18\alpha^3 - \alpha^4\right)\sqrt{1 + \alpha(34 + \alpha)} - 9 - 347\alpha - 682\alpha^2 + 138\alpha^3 + 35\alpha^4 + \alpha^5\right)}{(1 - \alpha)^3} < 0.$$

represented in Figure A.4. As shown, both contestants moderate their claims for any $\alpha > 0$ (L selects a policy higher than zero and R chooses a policy lower than one). This reduces the stake involved in the contest with respect to the NC-case and lowers the intensity of the conflict between lobbyists. This intensity decreases as the advantage of the contestant L increases. In the limit, the conflict would disappear because contestants would both claim policy 0.

Using (5), we obtain equilibrium efforts

$$\begin{aligned} x_L^l &= \frac{18\alpha}{2+\alpha\left(39+60\alpha+7\alpha^2\right)+\left(2+11\alpha+5\alpha^2\right)\sqrt{1+\alpha\left(34+\alpha\right)}} \quad \text{and} \\ x_R^l &= \frac{18\alpha}{7+\alpha\left(60+39\alpha+2\alpha^2\right)+\left(5+11\alpha+2\alpha^2\right)\sqrt{1+\alpha\left(34+\alpha\right)}}. \end{aligned}$$

For $\alpha \in (0,1)$, it can be shown that $x_R^l < x_L^l$ and $\lim_{\alpha \to 0} x_R^l = \lim_{\alpha \to 0} x_L^l = 0$.

Let us consider now the share contest. From (8) and (9) we get the equilibrium efforts as a function of the policy claims, $x_L^s\left(y_L, y_R; \alpha\right)$ and $x_R^s\left(y_L, y_R; \alpha\right)$.[5] Then, (3) and (4) can be used to obtain the contestants' indirect utilities:

$$\begin{aligned} V_L^s &= k_L - \left(\frac{x_L^s\left(y_L, y_R; \alpha\right)}{x_L^s\left(y_L, y_R; \alpha\right)+x_R^s\left(y_L, y_R; \alpha\right)}\left(y_L-y_R\right)+y_R\right)^2 - \alpha x_L^s\left(y_L, y_R; \alpha\right) \\ V_R^s &= k_R - \left(1-\frac{x_L^s\left(y_L, y_R; \alpha\right)}{x_L^s\left(y_L, y_R; \alpha\right)+x_R^s\left(y_L, y_R; \alpha\right)}\left(y_L-y_R\right)-y_R\right)^2 - x_R^s\left(y_L, y_R; \alpha\right). \end{aligned}$$

The analysis of these two functions will allow us to discover the optimal policy claims of the contestants. For the sake of exposition, we concentrate now on the symmetric case. When $\alpha = 1$, the equilibrium efforts in the contest are:

$$x_L^s\left(y_L, y_R; 1\right) = \frac{2y_R^2\left(1-y_L\right)\left(y_R-y_L\right)}{\left(1+y_R-y_L\right)^3} \quad \text{and}$$

$$x_R^s\left(y_L, y_R; 1\right) = \frac{2y_R\left(1-y_L\right)^2\left(y_R-y_L\right)}{\left(1+y_R-y_L\right)^3}.$$

The effect of a marginal concession by one contestant, say L, can be structured into three sub-effects: (i) a direct utility loss coming from supporting a less preferred policy, (ii) an indirect effort saving, and (iii) an indirect utility gain coming from a stronger influence (higher p_L) on the compromise policy. We next quantify these three impacts to study the overall effect of a concession by L on her own utility:

[5] See the Appendix for the full specification of $x_L^s\left(y_L, y_R; \alpha\right)$ and $x_R^s\left(y_L, y_R; \alpha\right)$.

(i) The direct utility loss is

$$-2\bar{y}\frac{x_L^s\left(y_L,y_R;1\right)}{x_L^s\left(y_L,y_R;1\right)+x_R^s\left(y_L,y_R;1\right)}dy_L=-2\left(\frac{y_R}{1+y_R-y_L}\right)^2dy_L.$$

Then, the magnitude of this effect when $y_R=1$ and L's concession is from the extreme policy $y_L=0$ is $-\left(1/2\right)dy_L$.

(ii) The indirect marginal effort saving is

$$-\frac{\partial x_L}{\partial y_L}dy_L=\frac{2y_R^2\left(1-y_R+y_R^2-y_L^2\right)}{\left(1+y_R-y_L\right)^4}dy_L.$$

When $y_L=0$ and $y_R=1$, this is equal to $\left(1/8\right)dy_L$.

(iii) The indirect utility gain coming from a change in the relative strength of the influence on the compromise policy is

$$\begin{aligned}&2\bar{y}\left(y_R-y_L\right)\left[\partial\left(\frac{x_L^s\left(y_L,y_R;1\right)}{x_L^s\left(y_L,y_R;1\right)+x_R^s\left(y_L,y_R;1\right)}\right)/\partial y_L\right]dy_L\\=\;&2\bar{y}\left(y_R-y_L\right)\frac{y_R}{\left(1+y_R-y_L\right)^2}dy_L.\end{aligned}$$

When $y_L=0$ and $y_R=1$, this indirect utility gain is $\left(1/4\right)dy_L$.

Therefore, in the symmetric case, the net effect of a marginal concession by L on her own utility is $\left(-1/2+1/8+1/4\right)dy_L=-\left(1/8\right)dy_L<0$. Meaning that contestant L does not have incentives to moderate her claim from zero. An analogous reasoning leads to the same conclusion for contestant R. Next proposition shows that this result extends to the asymmetric cases and that, additionally, $y_L=0$ and $y_R=1$ are actually the global maxima for L and R, respectively.

Proposition 2. *For $\alpha\in(0,1]$, the equilibrium policy claims in the lottery contest are $y_L^l>\tilde{y}_L$ and $y_R^l<\tilde{y}_R$, i.e. contestants moderate their policy claims. However, they do not concede in the share contest, i.e. $y_L^s=\tilde{y}_L$ and $y_R^s=\tilde{y}_R$.*

Proof. See the Appendix. □

This result is key to interpret the comparison between lottery and share contests in the C-case. Risk aversion explains why contestants will make concessions before the lottery contest but not before the share contest as the gains from reducing the uncertainty of the lottery contest are not present under the other contest design. This pushes contestants to strategically moderate their claims only in the lottery contest and this benefits them, as we will show next.

In terms of total effort, the concessions in the lottery contest widen the gap between the two contest designs compared to the NC-case. Under the C-case, the equilibrium effort exerted in the lottery contest is even lower than in the previous case, whereas the effort in the share contest does not vary because policy claims do not change. Consequently, aggregate effort is still lower in the lottery contest. With regard to individual efforts, the comparison between the two contest designs is unambiguous: The lower intensity of the conflict in the lottery contest involves a lower effort by each of the two contestants for any α. These comparisons are represented in Figure A.1 and summarized by the following result.

Lemma 2. *In the C-case, for $\alpha \in (0, 1]$:*

(a) Aggregate efforts in the share contest are larger than in the lottery contest.

(b) Both contestants exert more effort in the share contest.

Referring the comparison between equilibrium utilities, the concessions in the lottery contest reverse the results with respect to the NC-case (Proposition 1). Under that scenario, risk-averse contestants preferred the share contest (except by L for low values of α) because of the extra uncertainty of the lottery contest. Under the C-case, contestants strategically reduce this uncertainty by moderating their claims. This tilts the balance in favor of the lottery contest design. Specifically, both contestants are better-off in the lottery contest for most values of α (see Figure A.3).

Proposition 3. *In the C-case, for $\alpha \in (0, 1]$:*

(a) Contestants obtain a larger aggregate utility in the lottery contest.

(b) The disadvantaged contestant R obtains a larger equilibrium utility in the lottery contest except when her disadvantage is sufficiently large ($\alpha < 0.00748349$).

(c) The advantaged contestant L obtains a higher utility in the lottery contest.

Proof. See the Appendix. □

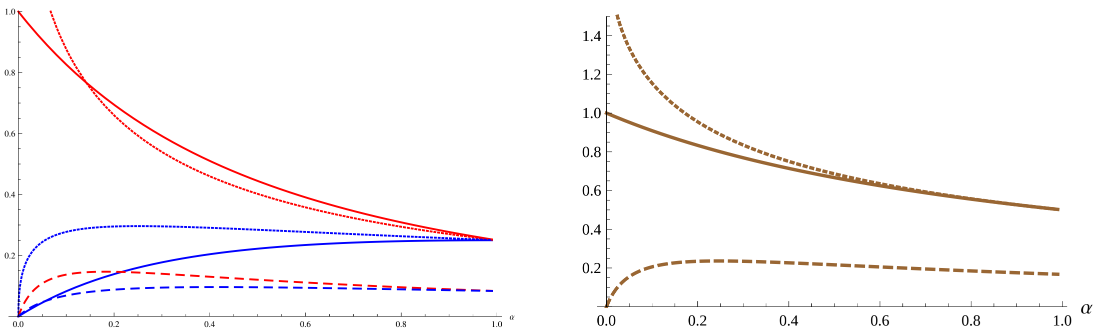
The intuitive explanation of this result relies again on the interplay among the three sub-effects introduced above: (i) the impact of the contest design on the expected/compromise policy, (ii) the uncertainty of the lottery contest, and (iii) the equilibrium effort costs under the two designs. In terms of effect (i), the possibility of commitment erodes the gap between the two contests designs with respect to the NC-case, as shown by Figure A.2(a). In the C-case, the lottery contest is less (more) favorable to the (dis)advantaged contestant L (R) than it was without commitment. The share contest still favors the disadvantaged contestant R (and punishes the advantaged contestant L), especially when this disadvantage is relatively marked (*i.e.* low values of α). Referring (ii), Figure A.2(b) shows how the strategic moderation of claims markedly reduces the uncertainty of the lottery contest. This makes this contest design more appealing for both contestants. The effect on the equilibrium effort costs follows the same line because the strategic moderation involves an effort saving in the lottery contest for both contestants. All these three sub-effects favor the lottery contest for L. For player R, only effect (ii) favors the share contest but this offsets the other two effects just for very low values of α.

4. Conclusion

This paper considers a lobbying process between two risk-averse contestants seeking to influence a regulator's policy choice that ranges over a continuum. In these situations, either tradition, law, political culture, or an organizer can frame this confrontation between two alternative settings: In one of them, the regulator's available choices are restricted to the discrete set of policies claimed by lobbyists. In the other, there is no restriction, so that the implemented policy can take any value on a continuous interval. We model these two settings as two different lobbying contests: In the lottery contest; the regulator selects the winner policy stochastically among the claims supported by lobbyists. Under this case, each lobbyist influences the winning probability of her policy claim. In the share contest, the implemented policy is deterministic, and it results from the balance of forces among lobbyists' efforts. Our paper provides each of these two types of contests with a micro-foundation based on equilibrium outcomes (either lobbying efforts or contestants' utilities) and the institutional framework; in particular, we consider a scenario in which lobbyists can only support their ideal policy in the contest and another in which they can credibly pre-commit to lobby for a different policy in the contest.

We show that the share contest involves a higher aggregate lobbying effort than the lottery contest under both institutional environments. Therefore, in situations where lobbying efforts are bribes for the regulator, this implies that a selfish regulator prefers share rather than lottery contests. The comparison in terms of lobbyists' utilities would be relevant where the contest designer (if any) cares about lobbyists or when lobbyists have some influence on the choice of the contest design. In this respect, our paper shows that, as expected, risk-averse lobbyists prefer the share contest as they fear the uncertainty of the lottery contest. However, this crucially depends on the institutional framework. If they can pre-commit to the policy they will lobby for in the contest, they will strategically moderate their policy claims only in the lottery contest. This moderation reduces the uncertainty of the lottery and calms down the conflict, reducing the equilibrium efforts exerted in the contest. These two effects switch the balance between lottery and share contests: the former is welfare enhancing under this institutional framework.

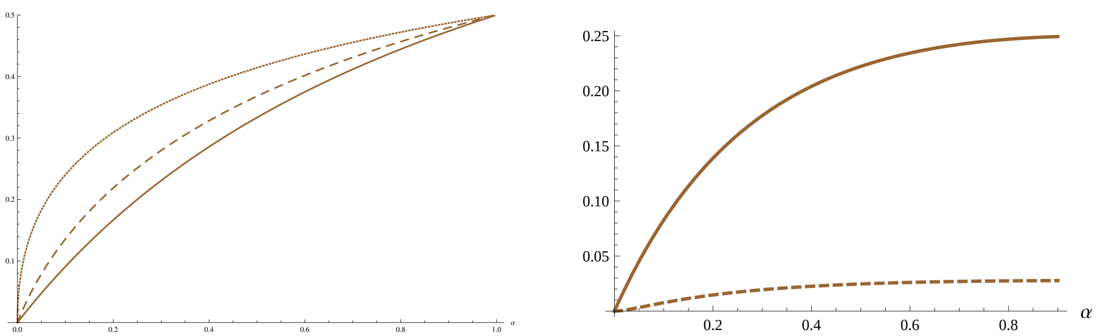
Appendix A. Figures



(a) Equilibrium efforts in the C-case (x_L^l: dashed red, x_L^s: dotted red, x_R^l: dashed blue, x_R^s: dotted blue) and in the NC-case (x_L^l: solid red, x_L^s: dotted red, x_R^l: solid blue, x_R^s: dotted blue).

(b) Total equilibrium effort in the C-case (lottery contest: dashed line, share contest: dotted line) and in the NC-case (lottery contest: solid line, share contest: dotted line).

Figure A.1: Individual and aggregate efforts with $k_L = k_R = 1$.



(a) Expected equilibrium and compromise policies. C-case: expected equilibrium policy in the lottery contest (dashed line) and compromise policy in the share contest (dotted line). NC-case: expected equilibrium policy in the lottery contest (solid line) and compromise policy in the share contest (dotted line).

(b) Variance of the two choices of the regulator in the lottery contest: C-case (dashed line) and NC-case (solid line).

Figure A.2: Policies and variance across contest designs.

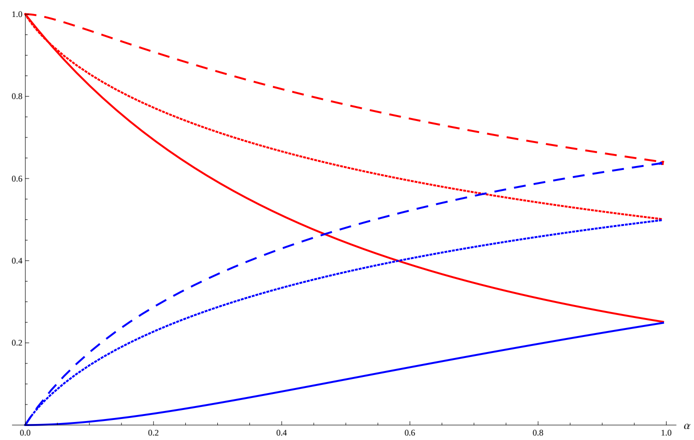


Figure A.3: Equilibrium utilities of the contestants in the C-case (V_L^l: dashed red, V_R^l: dashed blue, V_L^s: dotted red, V_R^s: dotted blue) and in the NC-case (U_L^l: solid red, U_R^l: solid blue, U_L^s: dotted red, U_R^s: dotted blue) for $k_L = k_R = 1$.

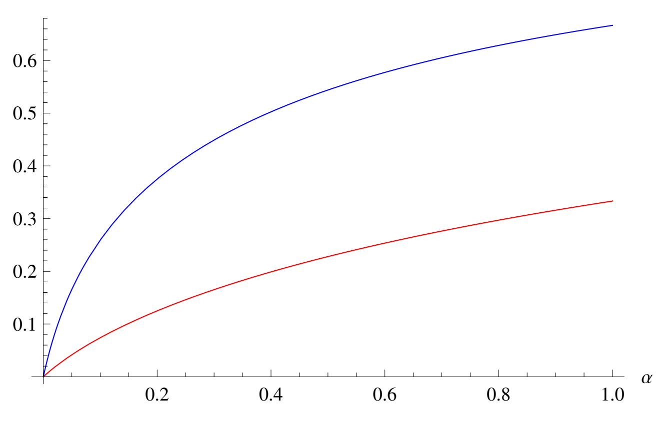


Figure A.4: Policy claims in the lottery contest (y_L^l: red, y_R^l: blue).

Appendix B. Proofs

Proof of Proposition 2

We proved results referring to the lottery contest in the main text. Under the share contest, equilibrium efforts as a function of y_L, y_R and α are

$$x_L^s = \frac{(H+y_L-\alpha(1-y_R))\left(-\alpha^2 y_R^3 - P_1(y_L,y_R,\alpha)\,y_R^2 + P_2(y_L,y_R,\alpha)\,y_R + P_3(y_L,y_R,\alpha)\right)}{2\alpha(1-y_L)(y_R-y_L)(1-\alpha)^3} \text{ and}$$

$$x_R^s = \frac{-\alpha^2 y_R^3 - P_4(y_L,y_R,\alpha)\,y_R^2 + P_5(y_L,y_R,\alpha)\,y_R + P_6(y_L,y_R,\alpha)}{(y_R-y_L)(1-\alpha)^3},$$

where $P_1 = 3 - H - y_L(1+2\alpha)$, $P_2 = H - y_L - \alpha(3-H-y_L)(1-2y_L) - \alpha^2 y_L^2$, $P_3 = \alpha(1-y_L)^2(H+y_L-\alpha)$, $P_4 = \alpha(3-H-y_L(1+2\alpha))$, $P_5 = H - y_L - \alpha(3-H-y_L)(1-2y_L) - \alpha^2 y_L^2.$, $P_6 = \alpha(H+y_L-\alpha)(1-y_L)^2$, and $H = \sqrt{y_L^2 - 2\alpha(1+y_R)\,y_L + 4\alpha y_R + (1-y_R)^2\alpha^2}$.

Inserting x_L^s and x_R^s into (3) and (4) we get the indirect utilities $V_L^s(y_L,y_R)$ and $V_R^s(y_L,y_R)$.

Examining the first derivative of $V_R^s(y_L,y_R)$ with respect to y_R we obtain that

$$\frac{\partial V_R^s(y_L,y_R)}{\partial y_R} = \frac{8\alpha^2(1-y_L)^2}{H+\alpha(1+y_R-y_L)+y_L(1-\alpha)}\left[H P_7(y_L,y_R,\alpha) + P_8(y_L,y_R,\alpha)\right] > 0.$$

Because, for $y_R \geq y_L$, $P_7(y_L,y_R,\alpha) = \alpha(1-y_L)(1-y_R) + y_L(1+y_R-2y_L) > 0$ and $P_8(y_L,y_R,\alpha) = \alpha^2(1-y_R)^2(1-y_L) + \alpha\left((2-y_L)(y_R-y_L)^2 + y_L(1-y_L)(2-y_L-y_R)\right) + y_L^2(1+y_R-2y_L) > 0$. Then $V_R^s(y_L,y_R)$ is maximized at $y_R^s = 1$.

The sign of the first derivative of $V_L^s(y_L,y_R)$ with respect to y_L at $y_R = 1$ is such that,

$$\left.\frac{\partial V_L^s(y_L,y_R)}{\partial y_L}\right|_{y_R=1} = \frac{8}{R+\alpha(2-y_L)+y_L(1-\alpha)}\left[P_9(y_L,y_R,\alpha)\,R + P_{10}(y_L,y_R,\alpha)\,y_L\right] < 0,$$

because $R = \sqrt{y_L^2 + 4\alpha - 4\alpha y_L} > 0$, $P_9(y_L,y_R,\alpha) = -(1-y_L)^3\alpha^2 - y_L^2(1-y_L)(3-y_L)\alpha - y_L^4 < 0$ and $P_{10}(y_L,y_R,\alpha) = -(5-3y_L)(1-y_L)^2\alpha^2 - y_L^2(1-y_L)(5-y_L)\alpha - y_L^4 < 0$. Then $V_L^s(y_L,y_R)$ is maximized at $y_L^s = 0$.

Proof of Proposition 3

Proving part (a) is immediate after showing parts (b) and (c). First, we prove these two parts for the symmetric case. Substituting $y_L^l = 1/3$ and $y_R^l = 2/3$ into the indirect utility functions (13) and (14) we obtain utilities $V_L^w(1/3,2/3) = V_R^w(1/3,2/3) = k_i - 13/36$, $i = L, R$. Instead, in the share contest, lobbyists do not moderate their claims. Thus, we obtain $V_L^s(0,1) = V_R^s(0,1) = k_i - 1/2$, $i = L, R$.

For the asymmetric case, $\alpha \in (0,1)$, inserting y_L^l and y_R^l displayed in (15) into (13) yields

$$\begin{aligned} V_L^l\left(y_L^l, y_R^l\right) &= k_L \\ &+ \frac{\alpha\left(12+203\alpha+138\alpha^2-33\alpha^3+4\alpha^4+\left(3\alpha^2-45\alpha-12\right)\sqrt{1+\alpha\left(34+\alpha\right)}\right)}{4\left(1-\alpha\right)^5}. \end{aligned}$$

For the share contest, the utility of L is given by (11),

$$V_L^s(0,1) = k_L - \frac{\alpha\left(3+\sqrt{\alpha}\right)}{\left(1+\sqrt{\alpha}\right)^3}.$$

For all $\alpha \in (0,1)$, $V_L^l\left(y_L^l, y_R^l\right) > V_L^s(0,1)$. Regarding contestant R, replacing y_L^l and y_R^l displayed in (15) into (14) yields

$$\begin{aligned} V_R^l\left(y_L^l, y_R^l\right) &= k_R \\ &- \frac{4+-33\alpha+138\alpha^2+203\alpha^3+12\alpha^4+\alpha\left(3-45\alpha-12\alpha^2\right)\sqrt{1+\alpha\left(34+\alpha\right)}}{4\left(1-\alpha\right)^5}. \end{aligned}$$

For the share contest, the utility of R is given by (12),

$$V_R^s(0,1) = k_R - \frac{\left(1+3\sqrt{\alpha}\right)}{\left(1+\sqrt{\alpha}\right)^3}.$$

It turns out that $V_R^l\left(y_L^l, y_R^l\right) = V_R^s(0,1)$ at $\alpha = 0.00748349$. For all $\alpha > 0.00748349$, $V_R^l\left(y_L^l, y_R^l\right) > V_R^s(0,1)$. It can be easily showed that both utilities are equal when $\alpha = 0$: $V_R^l\left(y_L^l, y_R^l\right) = V_R^s(0,1) = k_R - 1$ and $V_L^l\left(y_L^l, y_R^l\right) = V_L^s(0,1) = k_L$.

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