An Extended Theory of Rational Addiction

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Abstract

This study extends the Rational Addiction theory by introducing an endogenous discounting of future utilities. The discount rate depends on habits accumulation over time which occur because of the repeated consumption of an addictive good. The endogeneity of the discount rate affects consumption decisions via a habits dependent rate of time preference and discloses a patience-dependence trade off. The existence of a steady state in which habits do not grow and its optimality are proven. Local stability properties of the steady state reveal that the equilibrium can be a saddle node, implying smooth convergence to the steady state, but also a stable or unstable focus, potentially predicting real world behaviors as binge drinking or extreme addiction states that may drive to death. The stability of the steady state mostly depend on the habit formation process, suggesting that heterogeneity in habit formation may be a key component to explain heterogeneity in time preferences.

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The usual assumption in economics is that discount rates on future utilities are constant and fixed to each person, although they may differ between persons. This assumption is a good initial simplification, but it cannot explain why discount rates differ by age, income, education and other personal characteristics or why they change over time for the same individual, as when a person matures from being a child to being an adult.

(Becker G., "Accounting for Tastes," 1996)

1   Introduction

In Economics, consumption decisions are the result of an optimizing choice. The consumer chooses the consumption bundle that maximizes her felicity, represented by an utility function, given a budget constraint. Since Ramsey (1928), the dynamics of consumption are analyzed considering a representative consumer which maximizes the weighted sum of all her lifetime future utilities, where weights decrease as future gets far. In this context, the consumer is seen as a perfectly rational individual, who can exactly predict future earnings and prices and will never change her preference along time.

However, considering some peculiar goods, such as alcohol, tobacco or drugs, the phenomenon of addiction may arise and the Ramsey model could be inappropriate for a correct analysis. The state of addiction can lead the consumer to give more importance to present consumption of the addictive good respect to anything else.[1] However, addiction does not arise from one day to another, it is the result of an habitual consumption perpetuated over time. Consumption of possibly addictive goods implies a habit formation process, in which the habit is built day by day through consumption itself. Hence, a habit is formed when past and current consumption are linked by a positive relation. The higher is previous consumption, the larger the habit, and the higher should the current consumption level be to deliver the same utility. Pioneer studies on habits formation are due to Gorman (1967), Pollak (1970), Lluch (1974) and Boyer (1978), but the reference model, especially for applied studies is the Rational Addiction (RA) model proposed by Becker and Murphy (1988).

Following the RA approach, we assume that utility depends positively on current consumption and negatively on an index of habits. Utility reaches a peak after consumption rise to a permanently higher level, then it declines over time as the person becomes accustomed to that level. Comparisons with past consumption may be so effective that past consumption can be weighted more heavily than present consumption, and, when the habit formation process is sustained over time, a consumer may turn habits into a state of addiction. In other words, a habit may evolve into addiction by being exposed to the habit itself.[2] According to this approach, the habit formation process can be affected by economic variables and other exogenous factors such as demographic characteristics and the psychological state of consumers.

We propose an Extended theory of Rational Addiction (ERA) model which extends the RA model proposed by Becker and Murphy (1988) to allow for time-inconsistent consumers. Time consistency of consumers is an implication that derives directly from the choice of a constant discount rate for future utilities, which implies a constant rate of time preference.[3] The seminal works of Strotz (1956), Blanchard and Fisher (1989), Deaton (1992) and Romer (1986) have criticized the assumption of a constant rate of time preference as suggested more by convenience than economic rationales. Critics come also from empirical works, as Bishai (2004) and Laibson, Repetto, and Tobacman (2005), which rejects the hypothesis of a constant discounting.

Time consistency implies that the marginal rate of substitution between the consumption at time t and $t + 1$ is independent of time for each t different from t and $t + 1$. In fact, being the discount rate constant, the time at which the consumption choice is taken will not influence the choice itself. Suppose that in time $t = 0$ the consumer decide her optimal consumption path. If after 10 years she is allowed to reconsider the consumption path, being time-consistent, she will choose to stay on the path chosen in $t = 0$. Such a preference structure cannot properly describe situations such as addiction to alcohol

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[1]In general, addiction creates physical abstinence or withdrawal symptoms, when the use of the substance is discontinued, and generates tolerance, which is a physiological phenomenon requiring the individual to use more and more of the substance (Kennedy, 1987; Stein, Bentler, and Newcome, 1988). Tolerance for a substance may be independent of the drug ability to produce physical dependence which manifests itself by the symptoms of abstinence when the drug is withdrawn.

[2]Becker and Murphy (1988) define a person addicted to some goods when an increase in current consumption increases future consumption.

[3]The rate of time preference is a subjective indicator of impatience representing the desire of an agent to anticipate and enjoy the benefits stemming from higher current consumption. A high rate of time preference lowers the propensity towards future utility in determining current consumption choices.

(but also in drug use or cigarette smoking), or the existence of goods as holidays and works of art whose benefits continue over the consumption act.

To relax the hypothesis of time-consistent consumer we extend the RA model by assuming an endogenous discount rate depending on habits themselves. This approach, introduced by Epstein and Shi (1993), allows to have a non-constant rate of time preference which will depend on habits –an index of past consumption. Hence the consumption decision is influenced by the time at which the decision is taken, since different time periods imply different indexes of past consumption (which evolve along time) and hence different discount rate levels. Thus, a consumer that is allowed to reconsider her consumption plan in a future period may choose to change plans, because the accumulated habits have increased her discount rate, and hence the weights given to future utilities.

However, we keep the general utility structure of the RA model since it allows to model the negative effects that habits can have on consumers. For example, we may think at this negative effect as the impact that consumption of large quantities of the addictive goods may have on the general health status of the consumer (see Chaloupka, 1991; Dragone, 2009; Dragone and Savorelli, 2012; Chavas, 2013).

Our specification of the ERA model nests several models of habit formation and addiction,[4] such as:

- the Ramsey model (Ramsey, 1928; Koopmans, 1960; Cass, 1965), characterized by a constant rate of time preference;

- the Rational Addiction model of Becker and Murphy (1988), characterized by a constant discount rate and additive utility function with habits;

- the multiplicative habits model proposed by Carroll (2000), characterized by a constant discount rate and a multiplicative utility function with habits;

- the Uzawa (1968) or Obstfeld (1990) models, that assume an endogenous discount rate depending on current consumption, with no explicit modeling of habit formation;

- the Epstein and Shi (1993) model, characterized by an endogenous discount rate depending on habits.

With respect to the analysis of Becker and Murphy (1988) and Becker and Mulligan (1997), we extend the model in order to study the impact of habits on intertemporal consumption paths by the endogenous rate of time preference. One implication given by the endogenous discounting framework is that agents are no more assumed to be time-consistent. This result is in line with the critic of Gruber and Köszegi (2001) to the Rational Addiction model. This opens the way for modeling heterogeneity in the rate of time preferences through the process of habits accumulation.

The paper is structured as follows. Section 2 introduces the Extended Rational Addiction model, where the consumer maximizes a lifetime utility function with an endogenous discount rate depending on habits. Euler equations resulting from the ERA model are derived and analyzed. Section 3 proposes a definition of steady state and derives its properties. Section 4 assesses the stability properties of the steady state. Section 5 discusses heterogeneity in the habit formation process. Section 6 presents the comparative dynamics of the rate of time preferences and Section 7 concludes.

2 The Extended theory Rational Addiction (ERA)

The reference model in the field of addiction is the Rational Addiction by Becker and Murphy (1988). This model endows some characteristics that make it an appealing tool for applied works (see for example Chaloupka, 1991; Becker, Grossman, and Murphy, 1991, 1994), being characterized by a linear Euler equation with a simple test for the presence of addiction. This come at a cost: a constant discount rate equal to the rate of return to savings is assumed. This assumption has never been supported by theoretical reasoning or empirical evidence and has been widely criticized by the literature. We propose to relax this assumption in favor of an endogenous specification of the discount rate determined by the habits accumulated by the individuals.

The underlying assumption is that habits are likely to influence the discount rate and the rate of time preference inducing an increased impatience and, eventually, addiction. In doing so, we follow the work of Epstein and Shi (1993), in which the discount rate is a function of the *stock of habits*, an index of past

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[4]For a sketch of the proofs see Appendix A1.

consumption[5] denoted as z.[6] The degree of habits of an individual is represented by the stock variable z, which accumulates according to a dynamic process. Generally, this process depends on the personal characteristics of the individual and on the consumption level of the addictive good c. The endogeneity of the discount rate implies that the rate of time preference is also endogenous, making the analysis more interesting in terms of optimizing behavior.

In this framework, consumers maximize their felicity, which depends on the consumption of two goods and the stock of habits. The first good, g, is a composite good, which does not cause addiction. The second good, c, is a potentially addictive good,[7] which generates habits in consumption. We assume that felicity also depends on the strength of habits, synthesized by the stock of habits z and, in line with Becker and Murphy (1988), we assume that this effect is negative. Being a potentially addictive good, c generates habits over time, which in turn causes felicity to decrease, so that a larger amount of c needs to be consumed to maintain the same level of felicity.

To keep things simple, we assume that g does not interact directly with c and z in generating felicity, so that the utility function is additively separable and defined by $v\left(g\right) + w\left(c, z\right)$. In line with Cawley and Ruhm (2011), addiction is characterized by enforcement –marginal utility of current consumption increases with the stock of habits $(w_{cz} > 0)$[8]–, tolerance –the stock of habits lowers utility $(w_z < 0)$– and withdrawal –current consumption of the addictive good increases utility $(w_c > 0)$. Other usual regularity conditions impose $v_g > 0$, $v_{gg} \leq 0$, $w_{cc} \leq 0$, and $w_{zz} \geq 0$.

The endogenous specification of the discount rate closely follows Epstein and Shi (1993). In each instant in time, the discount rate is defined by a discount function $\theta\left(z\right)$ which depends on the stock of habits. The discount function $\theta\left(z\right)$ is a twice continuously differentiable function assumed to be strictly positive $(\theta\left(z\right) > 0)$, strictly increasing $(\theta_z\left(z\right) > 0)$ and concave $(\theta_{zz}\left(z\right) \leq 0)$.

Regarding the habits formation process, the literature proposes two approaches, which we call *partial adjustment* approach and *adaptive* approach.[9] The partial adjustment approach consists in considering the habit formation as an investment process. The stock of habits z accumulates as if it was capital. Investment is represented by current consumption and the stock of habits depreciates at rate σ.[10] The dynamic equation which describes this process is

$$\dot{z} = c - \sigma z. \tag{1}$$

In the adaptive approach, the habits accumulation process is due to the difference between current consumption of the potentially addictive good c and the current stock of habits z, through the rate of habits adjustment λ. If current consumption c exceeds the current stock of habits z, there will be formation of further habits, at the rate λ, otherwise, the stock of habits decreases. The dynamic equation which describes this approach is

$$\dot{z} = \lambda\left(c - z\right).$$

In this work we follow the partial adjustment approach, for consistency with the RA model. However, using the adaptive approach, as in (asin Epstein and Shi, 1993) would not alter substantially the results.

In the economic literature there is an open discussion if the endogenous discount rate should be considered increasing or decreasing with respect to consumption c. Koopmans (1960) suggests a decreasing rate of impatience, while Lucas and Stokey (1984) observe that an increasing rate of impatience is necessary to obtain a single, stable, non degenerate equilibrium point into wealth distribution in a deterministic horizon with a finite number of agents. According to Blanchard and Fisher (1989), the assumption of an increasing rate of impatience is difficult to defend *ex ante*. On the other side Epstein (1987a,b) argues that the more a person consumes, the more she discounts the future. In line with Epstein, we assume that the endogenous discount rate, $\theta\left(z\right)$, is strictly increasing with respect to the stock

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[5]Becker and Murphy (1988) refers to this concept as the "consumption capital," in a framework in which "past consumption of c affects current utility through a process of learning by doing... ." We prefer to use the definition proposed by Ryder and Heal (1973), which explicitly talks about habits in determining this index of past consumption.

[6]To enlighten notation we omit the time indication for time varying variables, except when integrating over time. For instance, $z(t)$ would generally be noted as simply z.

[7]When we say "potentially addictive" good, we mean that if that good is consumed under certain conditions (for example in high quantity), it generates addiction. Alcoholic beverages, for example, are potentially addictive goods. A moderate consumption of alcohol does not lead to addiction, but excessive consumption does.

[8]We denote the derivative of a generic function $f(x)$ with respect to some variable x $(\partial f(x)/\partial x)$ as $f_x(x)$. The second derivative $(\partial^2 f(x)/\partial x^2)$ is denoted as $f_{xx}(x)$.

[9]We propose these names to remind respectively the Partial Adjustment model and the Adaptive Expectation model, which have evident similarities with the habits specifications under discussion.

[10]As in Becker and Murphy (1988), "the rate of habits depreciation σ measures the exogenous rate of disappearance of the physical and mental effects of past consumption of c."

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of habits, and hence consumption. This assumption does not imply an always increasing discount rate. The discount rate may also decrease if, for instance, the consumer quits consuming the addictive good. In this case, the stock of habits z smoothly depreciates at rate σ, and consequently also the discount rate lowers.

This condition is necessary to ensure the stability of the long-run optimal consumption plan, because it guarantees that consumption levels in different dates are substitutes. In this case as wealth and consumption rise, the marginal private return to further savings, which depends on the marginal utility of future consumption, falls. A discount rate decreasing in consumption would cause consumption in different dates to be complements, so that an increase in present consumption rises the marginal utility of future consumption.

An implication of a discount rate strictly increasing with respect to consumption is that a higher consumption level at time t increases the discount rate applied to utilities after t. An increase in current consumption in t induces an increase in the rate of time preference: the consumer's desire to anticipate effects of future consumption is picked up by a larger consumption in $t+1$. An increase in current consumption in $t+1$ rises the stock of habits in $t+2$ inducing a further increase in the discount rate: the larger is previous consumption, the larger the habit, and the larger must be current consumption to deliver the same utility. Moreover, an increase in the discount rate rises the degree of adjacent complementarity and hence strengthens the commitment to all habits.

Defining a cumulative subjective discount rate Θ as[11]

$$\Theta = \int_0^t \theta\left(z(\tau)\right) - r \, d\tau, \tag{2}$$

the consumer problem can be written as

$$\max_{g(t),c(t)} \int_0^\infty \left(v\left(g(t)\right) + w\left(c(t), z(t)\right)\right) e^{-\Theta(t)} e^{-rt} dt \tag{3}$$

$$\begin{aligned}
s.t. \quad & \dot{a} = ra - g - pc, & & a(0) \geq 0 \quad \text{given} \\
& \dot{z} = c - \sigma z, & & z(0) \geq 0 \quad \text{given}, \\
& \dot{\Theta} = \theta\left(z\right) - r & & \Theta(0) = 0
\end{aligned}$$

where

$v\left(g\right)$ and $w\left(c, z\right)$	are the instantaneous utility functions
g	is current consumption of the non-addictive good
c	is current consumption of the addictive good
z	is the stock of habits
Θ	is the cumulative discount rate
a	is real wealth
p	is the relative price of the addictive good.[12]
r	is the rate of return to savings
σ	is the rate of habits depreciation,

and the rate of habits depreciation σ is assumed to be bounded between 0 and 1.

In (3) consumption goods g and c are control variables, while real wealth a, the stock of habits z and the cumulative subjective discount rate Θ are state variables. The optimization problem is solved according to the Maximum Principle of Pontriagin (Optimal Control theory), and the present value Hamiltonian function[13] is

$$H_d = e^{-\Theta}\left(v\left(g\right) + w\left(c, z\right)\right) + \tilde{q}\left(ra - g - pc\right) - \tilde{\varphi}\left(\theta\left(z\right) - r\right) + \tilde{\Psi}\left(c - \sigma z\right),$$

where $\tilde{q} = e^{rt}\hat{q}$, $\tilde{\varphi} = e^{rt}\hat{\varphi}$ and $\tilde{\Psi} = e^{rt}\hat{\Psi}$ are the discounted costate variables.

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[11]We use this formulation because it allows for some mathematical simplifications. This specification is equivalent to use the integral, from period 0 to t, of the discount function. In fact,

$$e^{-\Theta}e^{-rt} = e^{-\int_0^t (\theta(z(s))-r)ds}e^{-rt} = e^{rt - \int_0^t \theta(z(s))ds}e^{-rt} = e^{-\int_0^t \theta(z(s))ds}$$

[13]Note that the relation between current value and present value Hamiltonian is

$$H_d = e^{rt}H,$$

where H_d is the current value Hamiltonian, H is the present value Hamiltonian and r is the rate of return to savings.

The first order necessary conditions for an interior solution are[14]

$$\frac{\partial H_d}{\partial c} = 0 \longrightarrow \tilde{q}p = w_c e^{-\Theta} + \tilde{\Psi}$$

$$\frac{\partial H_d}{\partial g} = 0 \longrightarrow \tilde{q} = v_g e^{-\Theta}$$

and

$$\frac{\partial H_d}{\partial a} = r\tilde{q} - \dot{\tilde{q}} \quad \longrightarrow \quad \dot{\tilde{q}} = r\tilde{q} - r\tilde{q} = 0$$

$$\frac{\partial H_d}{\partial \Theta} = r\tilde{\varphi} - \dot{\tilde{\varphi}} \quad \longrightarrow \quad \dot{\tilde{\varphi}} = r\tilde{\varphi} - (v(g) + w(c,z)) e^{-\Theta}$$

$$\frac{\partial H_d}{\partial z} = r\tilde{\Psi} - \dot{\tilde{\Psi}} \quad \longrightarrow \quad \dot{\tilde{\Psi}} = (r+\sigma)\tilde{\Psi} + \tilde{\varphi}\theta_z - w_z e^{-\Theta}.$$

It is convenient to re-scale the co-state variables in order to eliminate Θ. Letting $q = \tilde{q}e^{\Theta}$, $\varphi = \tilde{\varphi}e^{\Theta}$ and $\Psi = \tilde{\Psi}e^{\Theta}$, the first-order necessary conditions become

$$q = \frac{1}{p}[w_c + \Psi] \tag{4}$$

$$q = v_g, \tag{5}$$

and, given that $q = \tilde{q}e^{\Theta}$, and

$$\dot{q} = \dot{\tilde{q}}e^{\Theta} + \tilde{q}e^{\Theta}\dot{\Theta} = 0 + q\dot{\Theta},$$

the remaining conditions are

$$\dot{q} = q(\theta(z) - r) \tag{6}$$

$$\dot{\varphi} = r\varphi - (v(g) + w(c,z)) \tag{7}$$

$$\dot{\Psi} = (r+\sigma)\Psi + \varphi\theta_z - w_z. \tag{8}$$

Differentiating equations (4) and (5) with respect to time we obtain

$$\dot{q} = \frac{1}{p}\left(w_{cc}\dot{c} + w_{cz}\dot{z} + \dot{\Psi}\right) \tag{9}$$

$$\dot{q} = v_{gg}g, \tag{10}$$

and using equation (10) with (6) we obtain the following Euler equation for $\dot{g}$

$$\frac{\dot{g}}{g} = \frac{v_g}{v_{gg}g}(\theta(z) - r) \tag{11}$$

Note that equating equations (4) and (5) it is possible to obtain a simple analytical expression for Ψ, which is

$$\Psi = pv_g - w_c. \tag{12}$$

The differential equation (7) gives a continuous time specification of the recursive structure of consumer preferences for every feasible consumption path. If we solve the differential equation[15] (7), we obtain

$$\varphi = \int_t^{\infty} (v(g(\tau)) + w(c(\tau), z(\tau))) e^{-\int_t^{\tau} \theta(z(s))ds} d\tau, \tag{13}$$

which is the present value of future utilities at time t and corresponds to the shadow price of the accumulated impatience rate Θ.

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[14]To save on notation, partial derivatives are denoted with a subscript. Thus, for example, $\partial w(c,z)/\partial c = w_c$.

[15]Recall that the solution for a differential equation with no constant coefficients as $\dot{y} + Py = Q$ is $y = e^{-\int P(t)dt}\int Q(t)e^{\int P(t)dt}dt + Ce^{-\int P(t)dt}$. The value that the solution approaches is referred to as the steady state so the limit for $t \to \infty$ of the solution is $y = \int Q(t)e^{\int P(t)dt}dt$.

By equating the two equations for $\dot{q}$, (6) and (9), we can solve for $\dot{c}$ and find the Euler equation

$$\frac{\dot{c}}{c} = \eta^c(g,c,z)\left(\rho^c(g,c,z,\varphi) - r\right),$$

where, by means of equations (1), (8), (5) and (12), the rate of time preference for good c is

$$\rho^c(\cdot) = \theta(z) - \frac{w_{cz}(c - \sigma z) + (r + \sigma)(pv_g - w_c) + \theta_z \varphi - w_z}{pv_g} \tag{14}$$

and the elasticity of intertemporal substitution is

$$\eta^c(\cdot) = \frac{pv_g}{w_{cc}c}.$$

Resuming, the resulting Euler equations for g and c are

$$\frac{\dot{g}}{g} = \frac{v_g}{v_{gg}g}(\theta(z) - r) \tag{15}$$

$$\frac{\dot{c}}{c} = \frac{pv_g}{w_{cc}c}\left(\theta(z) - \frac{w_{cz}(c - \sigma z) + (r + \sigma)(pv_g - w_c) + \theta_z \varphi - w_z}{pv_g} - r\right). \tag{16}$$

In the light of equation 13, the shadow price of the cumulative discount rate Θ may be seen as an index of impatience. In fact, being φ the weighted sum of all future utilities from time t, the higher future utilities are, the higher is patience, since the agent is willing to wait for the realization of his desires of consumption. For this reason, from now on we will refer to φ as the *rate of impatience*.

This formulation, that involves non separability of preferences, is suggested by Ryder and Heal (1973) who introduce the notion of adjacent complementarity.[16] An increase or decrease in consumption at $t-1$ can induce a variation of the marginal rate of substitution of current and future consumption at $t+1$. Complementarity is represented by a utility function that depends on both current consumption c and the stock of habits z, which is a weighted average of past consumption levels. Weights decline exponentially in the past at the exogenous depreciation rate σ, which can be interpreted as a measure of permanence of physical and mental effects of past consumption on present consumption c. As σ gets larger, less weight is given to past consumption in determining z.

The two Euler equations in (15) and (16) implicitly define the rates of time preference and the elasticities of inter-temporal substitution for the two goods. The rate of time preference for g is simply equal to the discount function $\theta(z)$, as for any standard inter-temporal model of consumption. However the discount rate is not constant and depends on the stock of habits z, implying that the rate of time preference for g depends on the past consumption of c. The inter-temporal elasticity of substitution for g is completely defined by the preference structure, and in particular on the shape of $v(g)$.

Proposition 1. *The rate of time preference for c, shown in equation (14), embeds*

    1. memory of past events through the stock of habits z and the rate of habits depreciation σ;

    2. perception of present events by the current consumption levels of c and g;

    3. the anticipation of future events by the present-value of future utilities φ.

Consumer behavior is non separable along time, revealing complementarity and time inconsistency. Present consumption of the potentially addictive good c and future consumption (represented by φ), depend on past consumption of the addictive good through the rate of habits depreciation σ, and need not to be valued equally along a locally constant consumption path. The rate of time preference expresses the propensity that a person reveals towards future utility in determining current choices. This depends on the ability to anticipate benefits of future consumption and the related physical and mental consequences of present and past consumption effects.

The Euler equation for c is different from canonical expressions mainly because it endows the complementarity between past consumption z, current consumption c and g, and future consumption by the rate of impatience φ through the endogenous rate of time preference and the elasticity of inter-temporal substitution.

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[16]Adjacent complementarity occurs when past consumption of a good raises the marginal utility of present consumption.

3 The Steady State

Given the Euler equations (15) and (16), the system of differential equations which describes the dynamic behavior of the ERA model is

$$\dot{z} = c - \sigma z \tag{17}$$

$$\dot{\varphi} = r\varphi - (v(g) + w(c,z)) \tag{18}$$

$$\dot{c} = \frac{pv_g}{w_{cc}} \left(\theta(z) - \frac{w_{cz}(c - \sigma z) + (r + \sigma)(pv_g - w_c) + \theta_z \varphi - w_z}{pv_g} - r \right) \tag{19}$$

$$\dot{g} = \frac{v_g}{v_{gg}} (\theta(z) - r) \tag{20}$$

$$\dot{a} = ra - g - pc. \tag{21}$$

Proposition 2. *Defining the steady state as an optimal solution to program (3) in which the stock of habits z and wealth a do not change over time[17] ($\dot{z} = 0$, $\dot{a} = 0$), a steady state for system (17-21) exists and lies on the optimal consumption path.*

Proof. If an equilibrium exists where $\dot{z} = 0$, it brings a number of consequences. First, if the stock of habits does not grow, then also consumption of the addictive good c does not grow at the steady state. In fact, from equation (17) we find that $c^* = \sigma z^*$.[18] Second, if z is constant also Θ is constant because of equation (26), and thus $\theta(z)$ is constant and equal to r, as implied by the cumulative discount rate motion equation and $\dot{\Theta} = 0$.

This considerations lead to a first important consequence. At the steady state the stock of habits z^* is uniquely determined by the discount function $\theta(z)$ and the interest rate r. Hence, the steady state consumption level c^*, which is determined by the stock of habits z^* and the rate of habits adjustment σ, also depend on the shape of the discount factor and the interest rate, but not on the structure of preferences.

The fact that c^* and z^* are constant implies that utility $w(c^*,z^*)$ is constant at the steady state, and so must be the index of impatience φ, which is the discounted value of the future streams of utilities. From equation (18), considering that $\dot{\varphi} = 0$, we obtain that

$$\varphi^* = \frac{v(g^*) + w(c^*,z^*)}{r}. \tag{22}$$

The fact that at the steady state $\theta(z) = r$ has another important consequence: g does not growth at the steady state. A condition for this to happen is that marginal utility v_g is constant, which by equation (5) implies that also q is constant, i.e. $\dot{q} = 0$. From condition (4) we find that

$$q^* = \frac{1}{p}(w_c + \Psi^*), \tag{23}$$

which in turn, by condition (5) defines the steady state value of g^*.

Finally, assuming that, at the steady state, real wealth a does not growth, from equation (21) we obtain the steady state value of wealth

$$a^* = \frac{g^* + pc^*}{r}.$$

So far we have determined the steady state levels of the variable involved in the dynamic system of the ERA model. To verify the existence of the steady state and that the steady state is an optimum solution to the maximization program we can use equation (19). We know that in the steady state $\dot{c}$ must be zero. For this to be verified the term in square brackets must also be zero, and being that $\theta(z^*) - r = 0$, we can concentrate on the term

$$w_{cz}^* (c^* - \sigma z^*) + (r + \sigma)\left(pv_g^* - w_c^*\right) + \theta_z^* \varphi^* - w_z^*.$$

Considering that, from (17) $(c^* - \sigma z^*)$ is 0 and substituting equation (12), we obtain

$$(r + \sigma)\Psi^* + \theta_z^* \varphi^* - w_z^*,$$

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[17] This definition comes from the consideration that it is physically implausible that a substance continues to generate habits to the infinite, otherwise, to keep a certain level of utility the consumer should continuously increase consumption of the addictive good. This in a word of finite resources is rather unlikely, although not impossible.

[18] With the superscript * we denote the variables at their steady state value.

and using equation (12), equation (8) can be set to 0, and

$$\Psi^* = \frac{w_z^* - \theta_z^* \varphi^*}{r + \sigma},$$

which implies that

$$(r + \sigma)\frac{w_z^* - \theta_z^* \varphi^*}{r + \sigma} + \theta_z^* \varphi^* - w_z^* = 0.$$

Then the steady state lies on the optimal solution to the ERA model. □

Proposition 3. *The steady state defined as ($\dot{z} = 0$, $\dot{a} = 0$) is unique.*

Proof. To verify the uniqueness of the steady state we consider the case in which $\dot{z}$ is still 0, but assume that this hypothetical steady state level of $z^\#$ is such that $\theta\left(z^\#\right) - r \neq 0$, i.e. $z^\#$ can be any value different from z^*. This implies that at the steady state the cumulative discount rate Θ constantly changes because $\dot{\Theta}$ is constant. By equation (20) also g constantly changes at the steady state.

Since z is constant, by equation (17) consumption of the addictive good c must be constant as well. The fact that g growth indefinitely at the steady state while c does not, arises the doubt that this hypothetical steady state may not satisfy all optimality conditions, so we need to verify that $\dot{c} = 0$ even when $\theta\left(z^\#\right) - r \neq 0$. The steady state levels of φ and Ψ are not constant and will change at constant rate proportional to $\dot{g}$. From (19), the necessary condition for $\dot{c} = 0$ is

$$\theta\left(z^\#\right) - r - \frac{w_{cz}^\# \left(c^\# - \sigma z^\#\right) + (r + \sigma)\left(pv_g - w_c^\#\right) + \theta_z^\# \varphi - w_z^\#}{pv_g} = 0,$$

but since $(pv_g - w_c) = \Psi$ and $\dot{\Psi} \neq 0$, using equation (11) the expression can be written as

$$\frac{pv_{gg}\dot{g} - (r + \sigma)\Psi - \theta_z^\# \varphi + w_z^\#}{pv_g},$$

which, by equation (20) is 0 if and only if

$$(r + \sigma)\Psi + \theta_z^\# \varphi = w_z^\# + \theta\left(z^\#\right) - r. \tag{24}$$

Note that the right hand side of equation (24) is constant, which would imply that either that both Ψ and φ were constant and take some specific values, which is not the case, or that

$$\Psi = \frac{w_z^\# + \theta\left(z^\#\right) - r - \theta_z \varphi}{r + \sigma},$$

which, again, is not the case. This implies that this alternative steady state is infeasible. □

It follows that if we define the steady state as an optimal solution to the optimization problem (3) in which the stock of habits is stationary ($\dot{z} = 0$), the unique feasible solution implies that the discount rate $\theta(z^*)$ has to be equal to the interest rate r.[19] This, in turns, defines the steady state value of the stock of habits z^* and consumption of the addictive good c^* purely as function of the habits generating process (through σ) and the discount function shape. This sounds intuitively reasonable, since a steady level of the stock of habits could be reached only if the process that generate habits stabilizes, and this does not involve preferences for the addictive good.

4   Local Stability Properties

The system described by (17-21) generates a 5 dimensional hyperplane which is divided into a number of regions ($2^5 = 32$, to be precise), each of which is characterized by a force leading the system toward the steady state or away from it.

We can reduce the dimension of the system noting that a is present only in the wealth accumulation equation. Since a does not influence other dynamic equations, we can consider the accumulation equation

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[19]Other steady states may exist when the definition of the steady state is different. However, at least we proved the existence and uniqueness of at least one steady state.

of wealth (21) as exogenous to the system and drop it from the stability analysis.[20] The new system generates a 4 dimensional hyperplane, which allows for an easier mathematical analysis.

Rewriting equation (19) as

$$\dot{c} = c\varepsilon^c(\cdot)\,(\rho(\cdot) - r)$$

where

$$\rho(\cdot) = \rho(g, c, z, \varphi)$$
$$\theta(z) - \frac{w_{cz}(c - \sigma z) + (r + \sigma)(pv_g - w_c) + \theta_z\varphi - w_z}{pv_g}$$
$$\varepsilon^c(\cdot) = \frac{pv_g}{w_{cc}c};$$

equation (20) as

$$\dot{g} = g\varepsilon^g(\cdot)\,(\theta(z) - r)$$

where

$$\varepsilon^g = \frac{v_g}{v_{gg}g};$$

and imposing each dynamic equation in to be equal to 0 at the steady state, the system can be written as[21]

$$z) \qquad 0 = c^* - \sigma z^*$$
$$\varphi) \qquad 0 = r\varphi^* - w(c, z) - v(g)$$
$$c) \qquad 0 = c^*\varepsilon^c(\cdot)\,(\rho(\cdot) - r)$$
$$g) \qquad 0 = g^*\varepsilon^g\,(\theta(z) - r).$$
$$a) \qquad 0 = ra^* - g^* - pc^*.$$

To analyze local stability first drop the $a)$ equation, since the corresponding eigenvalue is trivial and equal to r, and then we linearize the system by a first order Taylor expansion, obtaining

$$\dot{z} \cong c^* - \sigma z^* \tag{25}$$
$$\dot{\varphi} \cong r(\varphi - \varphi^*) - w_c(c - c^*) - w_z(z - z^*) - v_g(g - g^*)$$
$$\dot{c} \cong A(c - c^*) + B(g - g^*) + C(z - z^*) + D(\varphi - \varphi^*)$$
$$\dot{g} \cong E(g - g^*) + \theta_z(z - z^*)$$

where $A = (\varepsilon^c(\cdot) + c^*\varepsilon^c_c(\cdot))\,(\rho(\cdot) - r) + c^*\varepsilon^c(\cdot)\rho_c(\cdot)$, $B = c^*\varepsilon^c(\cdot)\rho_g(\cdot)$, $C = c^*\varepsilon^c(\cdot)\rho_z(\cdot)$, $D = c^*\varepsilon^c(\cdot)\rho_\varphi(\cdot)$ and $E = \varepsilon^g\,(\theta(z) - r) + g\varepsilon^g_g\,(\theta(z) - r)$. The resulting Jacobian matrix J is

	z	φ	c	g
z	$-\sigma$	0	1	0
φ	$-w_z$	r	$-w_c$	$-v_g$
c	C	D	A	B
g	θ_z	0	0	E

The stability properties of system (25) are determined by the eigenvalues of matrix J, through its characteristic polynomial

$$\alpha_0 + \alpha_1\lambda + \alpha_2\lambda^2 + \alpha_3\lambda^3 + \lambda^4,$$

where

$$\alpha_0 = r\theta_z B + v_g\theta_z D - E(rC + r\sigma A + \sigma w_c D + w_z D)$$
$$\alpha_1 = C(r + E) - AE(r - \sigma) + r\sigma(A + E) + D(\sigma w_c + w_z - w_c E) - \theta_z B$$
$$\alpha_2 = (r - \sigma)(A + E) + AE - r\sigma + w_c D - C$$
$$\alpha_3 = \sigma - r - A - E.$$

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[20]Being a recursively determined for each value of c and g by equation (21), when considered in the system it would imply an eigenvalue equal to r, which would not influence the stability properties of the system.

[21]Every function is intended as evaluated at the steady state.

The stability of the system is determined by the number of stable and unstable eigenvalues for matrix J, and in particular: if all eigenvalues are real and positive the steady state is an unstable point; if they are all real and negative, the steady state is a stable point; if they are all real but some are positive and some negative, the steady state is an saddle node; if some of them are complex, the steady state is a saddle focus.[22]

The roots of the characteristic polynomial correspond to the eigenvalues of matrix J. We know that an analytical solution exists for the roots of polynomials of the fourth degree, and that it is given by the Ferrari method. However the analytical solution and the relative stability analysis are by far too complex to be meaningful, even if we decided to use the Routh-Hurwitz stability criterion.

It is clear that the existence and stability of the steady state depends on the parameters values. For example, if the discount function $\theta(z)$ is always greater than r, a steady state is not possible because of the increasing property of $\theta(z)$. For this reason, to characterize the stability properties of the steady state we conduct a numerical simulation. The strategy of the simulation is as follows: first, we choose a functional form for the utility functions and the discount function; second, we chose plausible numerical values for the parameters of the system and compute the steady state; third, we let the parameters vary and calculate the numerical value of the eigenvalues of matrix J.

In the choice of the functional form of the utility function we depart from Becker and Murphy (1988). In fact, a quadratic utility function is characterized by a point of maximum and the non-satiation property, commonly assumed in consumption studies, would not hold. The model could still reach the steady state, if lower than the global maximum of utility, but this would imply an additional set of restrictions on the parameters which we prefer to avoid. Instead, we use logarithmic utility functions, and, for simplicity, a linear specification for the discount function:

$$\theta(z) = \theta_1 + \theta_2 z \tag{26}$$
$$w(c,z) = \beta \ln c - \gamma \ln z$$
$$v(g) = \alpha \ln g.$$

The discount function parameters are chosen to be both 0.02. Utility parameters α and β are set to 0.8, while γ is 0.2. The relative price of the addictive good p is fixed to 1.2, the rate of returns to savings r is equal to 0.05 and the rate of habits depreciation σ is 0.2. To evaluate the stability of the steady state we allow each parameter to vary.

The results of the simulations[23] indicates that the system is a saddle node in the neighbor of the parameters values. The eigenvalues are all real, three positive and one negative. This means that the system monotonically approaches the steady state, provided that the starting point lies on the stable eigenfold. The stability of the system respect to variations of the parameters is less trivial. For example, the steady state keeps being a saddle node for values of σ in the range $(0.1, 0.25)$. For values smaller than 0.1 or greater than 0.25 and smaller than 0.47 the equilibrium is an unstable focus, with a cyclical behavior that diverges from equilibrium. Then it becomes a stable focus and for values greater than 0.55 again a saddle node. Aside from the numerical values, that depend on the actual choices for the parameters and the consequent steady state's values, this means that the rate of habit depreciation is a key parameter for the behavioral interpretation of the model, and a fundamental ingredient to analyze heterogeneity in habit formation. Normal people with reasonable rates of habits depreciation can find an equilibrium in the consumption of the addictive substance and enjoy a moderate consumption. People with a small σ may be more likely to develop addiction and for them a stable equilibrium might not be possible. The cyclical fluctuations of the equilibrium for values in-between 0.25 and 0.55 could be interpreted as binge drinking, while very large rate of habits depreciation may not result in problematic behaviors.

Other parameters that could possibly be of interest from the behavioral point of view are the discount function parameters. In particular, θ_2 determines the degree of dependence of the discount rate on the stock habits z. The discount rate correspond to the rate of time preference of g and is one of the main determinants of that of c. We observe that for very small values of the parameter the system becomes an unstable focus. This seems reasonable since one of the equations that must be verified at the steady state is $\theta(z) - r = 0$. If $\theta(z)$ does not grow sufficiently fast with z, then the equilibrium could be unstable. On the other side, for any value larger than the chosen one, the system behaves as a saddle node.

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[22]The saddle node is the multidimensional equivalent of the saddle point. The system will converge toward the steady state only if the starting point lies on the stable eigenspace. In the saddle focus convergence (if the real part of the eigenvalue is negative) or divergence (if the real part of the eigenvalue is positive) will be cyclical.

[23]The table with all eigenvalues is available upon request, together with the program that generates the results.

Also the disutility caused by the stock of habits, through the utility parameter γ, plays an important role. We observe that the equilibrium becomes an unstable focus with a cyclical path if the disutility of habits is small (below 0.1). This may indicate that people with an insufficient perception of the health issues that consumption of alcohol, for example, may fall into binge drinking. For larger values the equilibrium is always a saddle node. Other parameters variations do not show unexpected results, and in general result in a saddle node equilibrium for wide intervals.

Similar result have been observed using isoelastic utility functions, while, as mentioned above, the case of a quadratic utility function is slightly more complicated. Our simulations suggest that the model behaves as in the previous cases in several situations, but, depending on the parameters of the utility function the steady state may be infeasible, since it can lie above the global maximum of the utility functions.

5 Heterogeneity in the Habits Formation Process

Heterogeneity is widely recognized as a necessary feature for almost any work on consumer demand. In fact it is a common opinion that each individual has its own preference structure and that a correct demand analysis should take this into account. When considering a dynamic context, heterogeneous preferences can be captured through the rate of time preference, as suggested by Fuchs (1982) and Lawrance (1991). Following this path, in this section we concentrate on introducing heterogeneity through the rate of time preference for c.

The definition of an endogenous rate of time preference permits separating the effects of heterogeneity from a generic habit effect. This is achieved through the definition of a rate of habits depreciation σ as a function of personal characteristics d. This imply that each individual develops habits and eventually addiction with a different consumption-habits path and achieve a different steady state. In particular, in Section 3 we have seen how the steady state level of the stock of habit z^* depends only on the discount function $\theta(z)$ and on the interest rate r. As a consequence, the steady state level of consumption of the addictive good c^*, which is equal to $\sigma(d)z^*$, depends on individual characteristics of the consumer d. Moreover, these characteristics may not be constant over time, as for age, which, as suggested by Bishai (2004), may influence the rate of time preference.

An example can help understanding our hypotheses. Define two categories of agents which compose the society, say myopic (m) and forward-looking (f)[24]

An agent is myopic if it is characterized by strong a preference towards actual consumption, with a large degree of impatience.

The impatience is caused by a large discount rate $\theta(z)$, which in turn is caused by a large stock of habits z. A large stock of habits may be caused by a rapid growth of z induced by a small rate of habits depreciation $\sigma(d)$.

Let us consider the case of a myopic agent. We may think, for example, at a middle-aged woman who becomes jobless in a certain moment of her life and has dependent children. This situation is one of the most frequent causes of "feminization" of alcohol consumption in Italy.[25] To escape her difficult family situation, she may occasionally take a drink. As time passes, if she is myopic, she builds habits, evolving into addiction when the stock of habits grows beyond some critical level.

An agent is forward looking if it is characterized by a preference towards future consumption, with a small degree of impatience. The discount rate $\theta(z)$ is small, due to a small stock of habits z, which remains small thanks to a large rate of habits depreciation $\sigma(d)$.

As a forward-looking agent, we may think about a thirty-years-old single, happily employed, who does not disdain a glass of wine per meal but he is a health-friend and a sport-lover. The likelihood that he reveals addiction is picked up by the rate of habits depreciation $\sigma^f(d)$ and his preferences towards alcohol consumption and habits. The larger is $\sigma(d)$, the less weight is given to past habits in determining current habits z. Therefore, even if the preference toward alcohol are strong, the large value of $\sigma^f(d)$ and the strong adversity against health injuries caused by habits, can avoid the arise of addiction.

Regarding the shape of the rate of habits depreciation $\sigma(d)$, to ensuring the needed regularity properties of the maximization problem it is necessary to bound it between 0 and 1. In fact, allowing for a negative rate of habits depreciation would rule out the existence of a steady state since habits would arise and continuously accumulate over time even if consumption of the addictive good happened just

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[24] Any resemblance between the symbols used to indicate the two kind of agents and those used to indicate males and females is purely coincidental.

[25] Italian Secretary of State for Health, On. Sirchia, Relazione sullo stato sanitario del paese 2001 - 2002.

once. On the other hand, for values of $\sigma(d)$ greater than 1 the stock of habits z would respond more than proportionally to a change in consumption c, allowing for a negative stock of habits z which would have no sense from a behavioral point of view. The limiting case of $\sigma(d) = 1$ implies that the stock of habits fully depreciates each period, thus excluding the possibility for habits to accumulate and for addiction to emerge.

Other characteristics of $\sigma(d)$, which are not relevant for obtaining an interior solution to the optimization problem, like slope and concavity, may have important implications for the behavioral analysis.

For example, one appealing feature could be an inverse-U-shape with respect to age, which would imply that an individual is more likely to develop addiction when young or elder, rather than when she is middle-aged. Another example is provided by recent medical evidence that suggests that men and women have different predisposition to addiction.[26] This may lead to a physiological differences in the process that generates addiction, in which women may be more exposed to the risk of become addicted. However, even if women need less alcohol than men to become alcohol addicted, they may have different preferences respect both to alcohol, which may be valued less, and to habits, regarded as the negative health effects that habits may bring, which could be valued more (in negative). The result can be a smaller probability of women to become addicted, since, even if women are more likely to become addicted for a given amount of alcohol consumed, they may be much less incline to consume respect to men.

6   Comparative Dynamics

The comparative dynamic analysis investigates how a variation of a variable or parameter of interest, such as c, z, φ and σ, affects the endogenous rate of time preference $\rho(g, c, z, \varphi)$.

The following propositions analyze the dynamic behavior of the rate of time preference with respect to the variables of interest in the neighborhood of the steady state. Proofs of these propositions are given in Appendix B.

Proposition 4. *The rate of time preference $\rho(g, c, z, \varphi)$ is strictly decreasing with respect to current consumption c.*

Proof. In the neighborhood of the steady state $c - \sigma z \approx 0$, thus the partial derivative of the rate of time preference with respect to current consumption is

$$\frac{\partial}{\partial c}\rho(\bar{g}, c, \bar{z}, \bar{\varphi}) = -\frac{1}{pv_g}\left(w_{cz} - (r + \sigma)w_{cc} - w_{zc}\right) = \frac{(r + \sigma)w_{cc}}{pv_g} < 0.$$

$\square$

The rate of time preference is decreasing with respect to an increase in current consumption of the possibly addictive good $c(t)$. This imply that the consumer is more concerned with immediate consumption rather than future consumption. In such cases there the need to "*save against a rainy day*" becomes less urgent and there is higher willingness to consuming today. In other words, a rise in current consumption causes an increase in the impatience level.

Proposition 5. *The rate of time preference $\rho(g, c, z, \varphi)$ is strictly decreasing with respect to the stock of habits z*

Proof. In the neighborhood of the steady state $c - \sigma z \approx 0$, thus the partial derivative of the rate of time preference with respect to the stock of habits is

$$\frac{\partial}{\partial z}\rho(\bar{g}, \bar{c}, z, \bar{\varphi}) = \theta_z - \frac{1}{pv_g}\left(-w_{cz}\left(r + 2\sigma\right) + \theta_{zz}\varphi - w_{zz}\right) > 0.$$

$\square$

Proposition 6. *The rate of time preference $\rho(g, c, z, \varphi)$ is strictly decreasing with respect to the rate of impatience φ.*

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[26] For example, a research conducted by Mancinelli, Vitali, and Ceccanti (2008) links the stronger effects produced by alcohol abuse, with a scarce presence of Alcohol Dehydrogenase (ADH), which is an enzyme involved in the metabolization of Alcohol. It is observed that, in general, women have smaller quantities of ADH than men.

Proof. The partial derivative of the rate of time preference with respect to the rate of impatience is

$$\frac{\partial}{\partial \varphi} \rho(\bar{g}, \bar{c}, \bar{z}, \varphi) = -\frac{\theta_z}{pv_g} < 0.$$

$\square$

An increase in the rate of impatience φ indicates that the consumer is less impatient, giving more weight to future consumption. Hence the rate of time preference is reduced and consumption of the addictive good grows at a smaller rate.

Proposition 7. *The rate of time preference $\rho(g, c, z, \varphi)$ is strictly decreasing with respect to the rate of habits depreciation σ if $w_c < pv_g$.*

Proof. In the neighborhood of the steady state $c - \sigma z \approx 0$, thus the partial derivative of the rate of time preference with respect to the rate of habits depreciation is

$$\frac{\partial}{\partial \sigma} \rho(\bar{g}, \bar{c}, \bar{z}, \bar{\varphi}) = -\frac{1}{pv_g} \left(pv_g - w_c\right) < 0 \quad iff \quad w_c < pv_g.$$

$\square$

An increase in the rate of habits depreciation implies that the rate of time preference declines, i.e. a decrease in the habit effects of the possibly addictive good tends to reduce the growth of consumption of c. This is true provided that the marginal utility of the possibly addictive good w_c is smaller than the marginal utility of the non addictive good, pv_g. Hence, unless preferences are biased towards the addictive good, an increase in the rate of habits depreciation lowers the rate of time preference, reducing the growth path of c.

The analysis of the variation of the rate of time preference respect to a variation of the stock of habits is more complex, with no meaningful results.

The analytical and behavioral properties of the rate of time preference allow us to describe the dynamic evolution of an agent from a condition of potential habit to a state of addiction.

Reconsider the case of the myopic and forward-looking agent introduced above. The degree of impatience of the myopic agent is generally higher (lower φ) than the degree of the forward-looking and so will be the degree of habits ($\sigma^m < \sigma^f$). The propensity to exchange current for future consumption becomes less and less considerable. The myopic agent reveals an increasing impatience since his stock of habits is larger.

The subjective rate of time preference of the myopic agent encloses reinforcement and tolerance, two behavioral factors that are closely related to the concept of adjacent complementarity. Reinforcement means to obtain the same level of utility consumption of the addictive good have to increase when current consumption increases, while tolerance means that given levels of consumption are less satisfying when past consumption has been greater.

The analysis reveals a patience-dependence trade-off. A patient person tends to have a lower stock of habits than an impatient person, since the desire to anticipate future consumption is lower. It is not surprising that addiction is more likely for people who discount the future heavily since they pay less attention to the adverse consequences. Becker, Grossman, and Murphy (1994) suggested that poorer and younger persons discount the future more heavily while Chaloupka (1991) found that less educated persons may have higher rates of time preference. The ability of anticipating the consequences of present and past consumption depends on income, education, rank and degree of awareness of dangers.

In line with Becker and Mulligan (1997) we find that *"... the analysis of endogenous discount rates implies that even fully rational utility-maximizing individuals who become addicted to drugs and other harmful substances or behavior are induced to place less weight on the future, even if the addiction itself does not affect the discount rate."*

7 Conclusions

Traditionally, the economic literature represents the structure of preferences in a dynamic context through utility functions discounted by a constant rate. This choice, often adopted for the sake of mathematical tractability, does not allow to explain situations where the discount rate changes over time for the same

individual (see Blanchard and Fisher, 1989; Deaton, 1992; Romer, 1986; Bishai, 2004; Laibson, Repetto, and Tobacman, 2005, amongothers).

Starting from the Rational Addiction model proposed by Becker and Murphy (1988) this paper develops an extension which allows for time-inconsistent consumers. Assuming an endogenous discount rate depending on past consumption as adopted in Epstein and Shi (1993), the study develops a formulation of inter-temporal preferences that generalizes several rational models of habit formation and addiction. The proposed rate of time preference supports a subjective structure of preferences that comprehends the memory of past events, the perception of present events and the anticipation of future events, revealing adjacent complementarity. The behavioral contents delivered by the ERA model are consistent with the results of the theory of Rational Addiction proposed by Becker and Murphy (1988) but introduce a further dimension in the analysis: the endogenous discount rate.

The proposed model presents a steady state in which the stock of habits, consumption of the possibly addictive and non-addictive goods, the shadow price of habits and the index of impatience do not grow. This steady state shows the interesting property that consumption level of the addictive good and the stock of habits are not determined by the corresponding preferences but just from the discount function parameters, the interest rate and the rate of habits depreciation.

Numerical simulations show that the steady state tends to be a saddle node, but some parameters variations, in particular the rate of habits depreciation, can introduce instability in the equilibrium through cyclical. This introduce a strong motivation for analyzing heterogeneity in the consumption of addictive goods, with the possibility of finding the conditions under which habits generate addiction or even a binge drinking behavior.

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A Proofs

Here we present a sketch of the proof that the ERA model is encompassing with respect to the models listed in the introduction. We show how imposing restrictions on the parameters or on the utility function the proposed model reduces to each of those models.

We start specifying the representative consumer optimization problem for the ERA model, i.e.

$$\max \int_0^\infty u(g(t), c(t), z(t)) e^{-\int_0^t \theta(z(\tau))d\tau} dt$$
$$s.t. \quad \dot{a} = ra - g - pc,$$
$$\dot{z} = c - \sigma z.$$

This specification is different from (3) in two things: the utility function that here is not divided intro the non-addictive and addictive parts, and the discount rate, which, as shown in footnote 14, is perfectly equivalent to the alternative specification.

Proof. **Epstein and Shi**. The ERA model reduces to the endogenous discounting model with habits proposed by Epstein and Shi (1993) by setting $u(g, c, z) = u(c)$ and assuming an adaptive habits accumulation process. Under these restrictions, the consumer maximization problem is

$$\max \int_0^\infty u(c(t)) e^{-\int_0^t \theta(z(\tau))d\tau} dt$$
$$s.t. \quad \dot{a} = ra - c,$$
$$\dot{z} = \lambda(c - z),$$

which corresponds to Epstein and Shi (1993, page 63 equation (2.1)). □

Proof. **Uzawa and Obstfeld**. The model proposed by Uzawa (1968) and Obstfeld (1990) is obtained by assuming that the utility function depends only on consumption of a non-addictive good, i.e. $u(g, c, z) = u(g)$, and that the endogenous discount rate depends on current consumption of the non-addictive good. This would be equivalent to set $\theta(z(t)) = \theta(g(t))$. The maximization program, taking into account that the habits accumulation equation must not be considered, becomes

$$\max \int_0^\infty u(g(t)) e^{-\int_0^t \theta(g(\tau))d\tau} dt$$
$$s.t. \quad \dot{a} = ra - c,$$

which correspond to Obstfeld (1990, pag 14, equations (20) and (21)). □

Proof. **Carroll**. The multiplicative habits model by Carroll (2000) is characterized by a constant discount rate, such that $\theta(z) = \theta$, by an utility function which depends on the ratio between consumption of the addictive good c and the stock of habits z, so $u(g, c, z) = u(c, z)$, and by an adaptive habits formation process. Even though the author explicitly sets up a CES like utility function from the beginning and uses a capital investment equation rather than a wealth equation, the following program

$$\max \int_0^\infty u(c(t), z(t)) e^{-\theta t} dt$$
$$s.t. \quad \dot{a} = ra - c,$$
$$\dot{z} = \lambda(c - z),$$

can be considered equivalent to Carroll (2000, page 7, equations (4) and(5)). □

Proof. **Rational Addiction**. The Rational Addiction model proposed by Becker and Murphy (1988) is characterized by constant discount rate and by an utility function which depends on current consumption of an undifferentiated good g, a possibly addictive good c and the stock of habits z. Hence the RA model can be easily be obtained by setting $\theta(z) = \theta$ and taking into account that the authors prefer to use a lifetime budget constraint rather than a motion equation for wealth. It is worth noting that the authors

initially consider an additional term, the expenditure on endogenous depreciation D, which is not present in the ERA model. Since the authors soon assume $D = 0$, we consider the program

$$\max \int_0^\infty u(g(t), c(t), z(t))e^{-\theta t} dt$$
$$s.t. \quad \dot{a} = ra - g - pc,$$
$$\dot{z} = c - \sigma z$$

to be equivalent to Becker and Murphy (1988, page 677, equationa (2),(3) and(4)). $\qquad\square$

Proof. **Ramsey**. The Ramsey model, often referred to as the basic consumption model, is characterized by a constant discount rate and a utility function depending on an undifferentiated good. Hence, we can recover it by assuming $u(g, c, z) = u(g)$ and $\theta(z) = \theta$. The resulting maximization problem is

$$\max \int_0^\infty u(g(t))e^{-\theta t} dt$$
$$s.t. \quad \dot{a} = ra - g,$$

which is one of the possible representations of the Ramsey model. $\qquad\square$