On the Efficient Provision of Public Goods by Means of Biased Lotteries: The Two Player Case

Jörg Franke[1] and Wolfgang Leininger[1]

[1]University of Dortmund (TU)
Department of Economics
Vogelpothsweg 87
44227 Dortmund
Germany

e-mail: Joerg.Franke@tu-dortmund.de
Wolfgang.Leininger@tu-dortmund.de

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Abstract

We provide a new approach to the free-rider problem in the context of public good provision based on a voluntary contribution game with a biased lottery. Neither extra resources, nor confiscatory taxes, nor other means of coercion are used in this setup. Instead, biasing the lottery in an appropriate way can induce the efficient amount of public good provision in equilibrium. We characterize the optimal combination of bias and lottery prize in the two player game, provide a lower bound on the respective lottery prize, and discuss the result and its limitation by example.

Key Words: Public good provision, biased lotteries, charities.

JEL classification: C72; D72; H41

1 Introduction

A standard problem in public economics is related to the observation that voluntary contributions to public good provision frequently result in inefficiently low levels of the public good due to free-riding of contributing agents. From a theoretical perspective free-riding occurs because contributing agents maximize their own payoffs without internalizing the positive externalities on other agents from public good provision. Hence, in voluntary contribution games there is typically inefficient underprovision of the public good in equilibrium, see for instance Bergstrom et al. (1986) for a seminal study.

Motivated by the frequently applied approach of charities in the real world, Morgan (2000) analyzed whether lotteries can be combined with voluntary contribution models to increase the provision of the public good. In his theoretical analysis he showed that specific types of lotteries (i.e., fixed prize raffles, where the respective prize sum is totally financed out of the lottery proceeds) induce negative externalities on participating agents which partially offset the positive externalities from public good provision. In the resulting equilibrium the net amount provided for public good provision (i.e., total lottery proceeds minus the prize sum) is therefore larger than in the original voluntary provision model without lottery such that the resulting allocation is a pareto-improvement. Furthermore, he showed that the amount of public good provision is increasing in the prize sum. These results also proved to be robust with respect to various modifications and extensions of the underlying model, compare Duncan (2000), Maeda (2008), Pecorino and Temimi (2007), Lange (2006), Goeree et al. (2005), and Faravelli (2011). However, a caveat of most of these studies, as well as of the original contribution of Morgan (2000), is that no prize sum of finite value can induce the efficient allocation: While there are pareto-improvements in comparison to the original voluntary contribution model, there is still inefficient underprovision of the public good in equilibrium.

We explicitly address this issue in our paper and show for the two player case that an appropriately biased lottery with a *finite* prize sum can induce the efficient level of public good provision if agents are not identical.[1] More precisely, we characterize feasible combinations of prize sum and bias that induce the efficient level of public good provision in any interior equilibrium of the respective voluntary contribution game. These efficient combinations of prize sum and bias are not unique, but there exists a close relation between the two instruments. Based

[1]The underlying intuition stems from Franke et al. (2013), where we analyzed the revenue-enhancing potential of optimally biased contest games and showed that biasing the contest success function increases total contributions by the contestants. Whether biasing the lottery in a public good framework could be sufficient to induce efficient public good provision is not obvious ex-ante.

on this relation we derive the minimal prize sum that is (in combination with the corresponding bias) necessary to induce the efficient level of public good provision for a given degree of heterogeneity among the players. While the relation between the minimal prize sum and the efficient provision level of the public good is decreasing in the heterogeneity of the consumers, we also show by example that the absolute value of the minimal prize sum is non-monotonic in the underlying heterogeneity. Hence, a specific degree of heterogeneity would be best (in the sense that it requires the lowest prize sum) to induce the efficient amount of public good provision. Moreover, we use the example to explicitly address the issue of (interior) equilibrium existence and participation constraints. Here, a sufficient degree of heterogeneity guarantees that optimal bias-prize combinations result in interior equilibria with efficient public good provision where both players voluntarily decide to contribute positive amounts to the public good.

Our paper is related to some recent studies that also use modified lotteries to induce the efficient level of public good provision, see Kolmar and Wagener (2012) as well as Giebe and Schweinzer (2013). However, both papers rely on coercive taxation to finance the lottery prizes. In contrast, we retain the original assumption of Morgan (2000), where lottery prizes are completely financed out of lottery proceeds, which makes our analysis more applicable in the context of charities and other non-public organization that typically lack tax power.

The rest of the paper is organized as follows. In the next section we formally introduce the general model and summarize previous results from the existing literature. In section 3 we analyze the two player case and show that efficient combinations of bias and lottery prize exist. Moreover, we establish the existence of a minimal prize sum that is necessary to induce the efficient provision level of the public good. We clarify the implications and limitations of our results with the help of an example in section 4 and conclude in section 5 by discussing robustness and possible extensions.

2 The Model

There are n risk-neutral consumers indexed by $i = 1, \dots, n$ with quasi-linear utility functions of the type $u_i(w_i, G) = w_i + h_i(G)$, where w_i is the wealth of consumer i and G denotes the level of the public good provided. It is standard to assume that $h_i^{'} > 0$ and $h_i^{''} < 0$ for all i. We also assume that $h_i^{'}(0) > 1$ for all i, which implies that the public good is socially desirable. Wealth can be transformed into the public good using the production function $f(w) = w$ such that one unit of (private) wealth can be transformed into one unit of the public good.

In this quasi-linear framework the efficient amount $\hat{G}$ of the public good maximizes the utilitarian welfare function $W = \sum_{i=1}^{n} u_i(w_i, G)$, where we assume that wealth constraints are non-binding. This results in the famous optimality condition due to Samuelson (1954):

Proposition 2.1 *The efficient amount of public good $\hat{G}$ is characterized as the solution to the equation $\sum_{i=1}^{n} h_i'(\hat{G}) = 1$.*

A voluntary contribution game is defined as a non-cooperative game where consumers of the public good maximize individual utility by deciding individually their contribution level to public good provision. Bergstrom et al. (1986) prove that the resulting Nash equilibrium exists and is unique. Moreover, they show that the amount G^{BBV} of the public good that is provided in the Nash equilibrium of the voluntary contribution game is always below the efficient level $\hat{G}$. Intuitively, inefficient underprovision can be attributed to the fact that utility-maximizing consumers do not internalize the positive externalities from public good provision; that is, they freeride on each other.

Proposition 2.2 *The Nash equilibrium of the voluntary contribution game leads to underprovision of the public good: $G^{BBV} < \hat{G}$.*

Morgan (2000) modified the voluntary contribution game by introducing fixed-prize raffles, where a pre-announced fixed prize of value R is offered by the prospective provider of the public good. Individual contributions to the public good, from now on denoted by x_i, are awarded with lottery tickets on a one-to-one basis and the prize is given to the lucky buyer of the winning lottery ticket. Hence, agent i's probability to win the prize is governed by its relative amount of lottery tickets $\frac{x_i}{\sum_{j=1}^{n} x_j}$ (as long as $\sum_{j=1}^{n} x_j > 0$), which is a special case of a so-called contest success function introduced by Tullock (1980). The prize sum R itself is financed out of total contributions $\sum_{i=1}^{n} x_i$, which implies that only the remaining amount $\sum_{i=1}^{n} x_i - R$ can be used to finance the public good. Consumer i consequently maximizes its expected utility:

$$u_i(x_i, x_{-i}) = w_i - x_i + \frac{x_i}{\sum_{j=1}^{n} x_j} R + h_i\left(\sum_{j=1}^{n} x_j - R\right).$$

For this modified voluntary contribution game involving lotteries Morgan (2000) proves that the net amount G^M of public good provision (total contributions minus prize sum) in the unique equilibrium is actually higher than in the original voluntary contribution game. Intuitively, the positive externalities from public good provision are partially balanced by the negative externalities from the lottery such that the increase in total contributions is not only sufficient to finance the lottery prize but additionally leads to even higher remains to finance the public good. Hence, there is a pareto-improvement with respect to the original voluntary contribution game without lottery. However, underprovision of the public good still prevails in equilibrium because no finite prize sum R leads to the efficient amount $\hat{G}$ of the public good.

Proposition 2.3 *The Nash equilibrium of the voluntary contribution game with lotteries leads to a pareto-improvement with respect to the original voluntary contribution game; however, for any finite prize sum underprovision of the public good prevails:* $G^{BBV} < G^{M} < \hat{G}$.

3 Biased Lotteries: The Two Player Case

We consider a voluntary contribution game with two consumers and biased lotteries, where each individual amount x_i of bought lottery tickets is weighted by a parameter $\alpha_i > 0$ before the final winner is determined. The resulting winning probability for consumer $i = 1, 2$ is therefore biased: $p_i = \frac{\alpha_i x_i}{\alpha_1 x_1 + \alpha_2 x_2}$. As this contest success function is homogeneous of degree zero, we can normalize it such that only lottery tickets of the second player are weighted by a factor $\alpha > 0$. This normalization leads to the following expected utility functions:

$$u_1(x_1, x_2) = w_1 - x_1 + \frac{x_1}{x_1 + \alpha x_2} R + h_1 (x_1 + x_2 - R) \tag{1}$$

$$u_2(x_2, x_1) = w_2 - x_2 + \frac{\alpha x_2}{x_1 + \alpha x_2} R + h_2 (x_1 + x_2 - R) \tag{2}$$

For this modification of the Morgan (2000) setup existence and uniqueness of the equilibrium are preserved. Moreover, in the following we assume that the equilibrium in the two player game is interior[2] and can therefore be characterized by the respective first order conditions (second order conditions for a maximum always hold):

$$-1 + \frac{\alpha x_2^*}{(x_1^* + \alpha x_2^*)^2} R + h_1' (x_1^* + x_2^* - R) = 0, \tag{3}$$

$$-1 + \frac{\alpha x_1^*}{(x_1^* + \alpha x_2^*)^2} R + h_2' (x_1^* + x_2^* - R) = 0. \tag{4}$$

Our objective is to show that there exist bias-prize combinations (α, R) such that the resulting equilibrium allocation induces the efficient level $\hat{G}$ of public good provision. Formally, eq. (3) and (4) must hold simultaneously with the Samuelson condition from Proposition 2.1, where

[2]In section 4 we discuss this assumption and its implications in detail.

total contributions must finance the prize sum of the lottery:

$$h_1'(\hat{G}) + h_2'(\hat{G}) = 1 \Longleftrightarrow x_1^* + x_2^* = \hat{G} + R. \tag{5}$$

Combining eqs. (3) – (5) leads to the following system of equations:

$$\frac{\alpha(x_1^* + x_2^*)}{(x_1^* + \alpha x_2^*)^2} R = 1, \tag{6}$$

$$x_1^* + x_2^* = \hat{G} + R. \tag{7}$$

The next two results show that heterogeneity among the two players is a necessary condition for the existence of a bias-prize combination that induces the efficient allocation in equilibrium. We proof this by contradiction and show that for identical players biased lotteries can never induce the efficient level of public good provision in equilibrium.

Lemma 3.1 *The system of equations in (6) and (7) does not have a symmetric solution* $x_1^* = x_2^*$.

Proof. Let $x_1^* = x_2^*$. Then eq. (6) and (7) can be simplified to $\hat{G} = -\frac{(1-\alpha)^2}{(1+\alpha)^2} R \leq 0$. The last inequality is a contradiction because the public good is socially desirable ($\hat{G} > 0$). □

An important corollary to the above Lemma is the following result.

Lemma 3.2 *Identical consumers will not provide the efficient amount* $\hat{G}$ *of the public good in any voluntary contribution game with biased lotteries independently of the respective bias-prize combination* (α, R).

Proof. For identical consumers the two first order conditions in eq. (3) and (4) are identical which implies that $x_1^* = x_2^*$. Application of Lemma 3.1 then leads to the result. □

In the following we therefore concentrate on non-identical consumers. It is convenient to define the following measure of heterogeneity:

Definition 3.3 *Two consumers are heterogeneous if* $h_1'(\hat{G}) \neq h_2'(\hat{G})$. *Heterogeneity is measured by parameter* $\hat{h} = \frac{1-h_2'(\hat{G})}{1-h_1'(\hat{G})}$.

It should be mentioned that this measure only depends on the preference parameters (because they determine the efficient level $\hat{G}$ of public good provision). Moreover, the measure is always positive ($\hat{h} > 0$), which is implied by eq. (3) and (4).

The main result of the paper is presented in the following theorem which states that as long as consumers are non-identical and the equilibrium is characterized by first order conditions, there exist feasible combinations of bias and prize sum that induce the efficient level $\hat{G}$ of public good provision.

Theorem 3.4 *If consumers are heterogeneous (i.e. $\hat{h} \neq 1$) there always exist feasible combinations (α, R) of lottery prize sum and bias such that equations (6) and (7) are simultaneously satisfied.*

Proof. From eq. (3) and (4) it is obvious that $x_1^* = \hat{h} \cdot x_2^*$. In combination with eq. (7) this expression can be used to explicitly solve for efficient equilibrium contributions x_1^* and x_2^*:

$$x_1^* = \frac{\hat{h}}{\hat{h}+1}(\hat{G}+R), \qquad x_2^* = \frac{1}{\hat{h}+1}(\hat{G}+R). \tag{8}$$

Plugging this in (6) and solving for $\hat{G}$ leads to:

$$\hat{G} = \frac{(\alpha-1)\left(\hat{h}^2-\alpha\right)}{\left(\hat{h}+\alpha\right)^2}R. \tag{9}$$

The right hand side of eq. (9) only depends on the heterogeneity measure $\hat{h}$ and the two instruments α and R. Obviously, this equation can always be satisfied with a sufficiently large R provided that the factor in front of R is positive: $(\alpha-1)\left(\hat{h}^2-\alpha\right) > 0$. Observe, that both expressions in brackets are positive if $1 < \alpha < \hat{h}^2$, and both expressions are negative if $\hat{h}^2 < \alpha < 1$. Moreover, $\hat{h} \neq 1$ because consumers are assumed to be heterogeneous. Consequently, for any $\hat{h} \neq 1$ an $\alpha > 0$ exists such that $(\alpha-1)\left(\hat{h}^2-\alpha\right) > 0$ holds; and for this α, in turn, there exists $R > 0$ such that eq. (9) holds. □

Theorem 3.4 and its proof imply that there exists a continuum of feasible and efficient (α, R)-combinations. In fact, eq. (9) provides the exact relation between the prize sum and the respective bias. We use this relation to derive the unique minimal prize sum R_{min} that (in combination with the respective bias) is necessary to induce the efficient amount of public good provision. The next proposition also shows that this minimal prize sum R_{min} depends on the underlying heterogeneity of the consumers in the following sense: If the degree of heterogeneity among consumers increases then the relation between the minimal prize sum and the efficient provision level $\hat{G}$ decreases. This negative relation can be attributed to the fact that under higher heterogeneity the bias will become more effective than the prize sum in inducing additional contributions by the consumers.

Proposition 3.5 *Consider a voluntary contribution game with biased lotteries and heterogeneous consumers. There exists a prize sum* $R_{\min} > 0$ *with the following properties:*

i) Let (α, R) *be a bias-prize combination such that equations (6) and (7) are simultaneously satisfied. Then* $R \geq R_{\min}$ *must hold.*

ii) For any $R \geq R_{\min}$ *there exists a bias* α *with* $0 < \alpha \leq \alpha_{max}$ *such that equations (6) and (7) are simultaneously satisfied.*

iii) The relation $\frac{R_{\min}}{\hat{G}}$ *is monotonically decreasing in the heterogeneity of the consumers.*

Proof. Combining eqs. (6) and (7) leads to eq. (9) that we rewrite as $\hat{G} = f(\alpha) \cdot R$, where $f(\alpha) = \frac{(\alpha-1)(\hat{h}^2-\alpha)}{(\hat{h}+\alpha)^2}$. Note first that $f'(\alpha) = \frac{(\hat{h}-\alpha)(\hat{h}+1)^2}{(\hat{h}+\alpha)^3}$, which implies that $f'(\alpha) > 0$ for $\alpha < \hat{h}$ and $f'(\alpha) < 0$ for $\alpha > \hat{h}$. Hence, $f(\alpha)$ is pseudo-concave (single-peaked) and the unique maximum of f occurs at $\alpha_{max} = \hat{h} > 0$. Moreover, $f(\alpha)$ attains its maximum in the interval $\left[1, \hat{h}^2\right]$ if $\hat{h} > 1$, and in the interval $\left[\hat{h}^2, 1\right]$ if $\hat{h} < 1$. The maximand $\alpha_{\max} = \hat{h}$ gives the maximum of $f(\alpha)$ which is $f(\alpha_{\max}) = \frac{(\hat{h}-1)^2}{4\hat{h}} > 0$. As $f(\alpha) \cdot R = \hat{G}$, the prize sum R must exceed $R_{\min} = \frac{\hat{G}}{f(\alpha_{\max})}$ which implies that $R \geq R_{\min} = \frac{4\hat{h}}{(\hat{h}-1)^2} \cdot \hat{G} > 0$. This establishes part (i) of Proposition 3.5.
Part (ii) of the Proposition holds because $f(a)$ is pseudo-concave and can be varied continuously between 0 and $f(\alpha_{max})$.
For part (iii) define $R_{\min} = g\left(\hat{h}\right) \cdot \hat{G}$, where $g\left(\hat{h}\right) = \frac{4\hat{h}}{\left(\hat{h}-1\right)^2}$. Note that $g'\left(\hat{h}\right) = -\frac{4\left(\hat{h}+1\right)}{\left(\hat{h}-1\right)^3}$, which implies that $g\left(\hat{h}\right)$ is monotonically increasing in $\hat{h}$ for $\hat{h} < 1$ and monotonically decreasing in $\hat{h}$ for $\hat{h} > 1$ with a pole at $\hat{h} = 1$. □

Proposition 3.5 (iii), however, does not imply that the absolute value of the minimal prize sum R_{min} is monotonic in the heterogeneity of the consumers. An increase in $\hat{h} > 1$, for instance, also implies that $\hat{G}$ will increase. Hence, $R_{\min} = g\left(\hat{h}\right) \cdot \hat{G}$ could still behave non-monotonically. The example in the following section shows that this indeed can occur. Moreover, the example is useful to clarify the issues of participation constraints and interiority of equilibria.

4 An Example

We consider a voluntary contribution game with biased lotteries where the idiosyncratic utility from public good consumption is specified as follows: $h_1(G) = b\sqrt{G}$ with $b \geq 0$ and $h_2(G) = \sqrt{G}$. For this specification the efficient amount of public good provision is $\hat{G} = \frac{(1+b)^2}{4}$ (based on Proposition 2.1), while the measure of heterogeneity reduces to $\hat{h} = b$. The minimal prize sum in this case is $R_{\min}(b) = \frac{b(1+b)^2}{(b-1)^2}$. Analyzing this expression leads to the following result.

Proposition 4.1 *The minimal prize sum $R_{\min}$ need not be monotonic in the heterogeneity of the consumers.*

Proof. Note that $R'_{\min}(b) = \frac{(b+1)(b^2-4b-1)}{(b-1)^3}$ is negative for $b \in (1, 2+\sqrt{5})$, and positive for $b \in (0,1) \cup (2+\sqrt{5}, \infty)$. This implies that $R_{\min}(b)$ is non-monotonic in b. □

Intuitively, the shape of function $R_{\min}(b)$ reflects the derived results: The pole at $b = 1$ implies that no finite prize can induce the efficient amount of public good provision if players are identical (Lemma 3.1 and 3.2). Hence, a minimal degree of heterogeneity is necessary to be able to generate the efficient outcome with biased lotteries (Theorem 3.4). Moreover, the bias is more effective for higher degrees of heterogeneity which implies that the minimal prize sum can be reduced accordingly. Thus, R_{min} is increasing for $b \in (0,1)$ and decreasing for $b \in (1, 2+\sqrt{5})$. For very large degrees of heterogeneity ($b > 2+\sqrt{5}$), however, the bias becomes less effective such that the minimal prize must increase as well to compensate the lower effectiveness of the bias; that is, R_{min} is increasing for $b > 2+\sqrt{5}$ and therefore non-monotonic (Proposition 4.1). This also implies that the lowest necessary prize sum that guarantees the efficient amount of public good provision is located at an intermediate level of heterogeneity ($b = 2+\sqrt{5} \approx 4.236$).

Participation Constraints

In our previous analysis we assumed that equilibrium contributions are characterized by first order conditions; that is, participation constraint are satisfied for both consumers in the efficient equilibrium. This assumption is generally satisfied in biased lottery contest games with two players (without public good component) and also in the unbiased Morgan setup with sufficiently high prize sum, see Franke et al. (2013) and Morgan (2000). In voluntary contribution games with biased lotteries this assumption is less innocuous, as we now show based on the previous example. We proceed by assuming that efficient equilibrium contributions are characterized by first order conditions and then analyze the respective participation constraints in equilibrium.

Efficient equilibrium contributions are characterized in eq. (8) and can be expressed as:

$$x_1^* = \frac{\alpha b(b+1)^3}{4(\alpha-1)(b^2-\alpha)}, \qquad x_2^* = \frac{\alpha(b+1)^3}{4(\alpha-1)(b^2-\alpha)}. \tag{10}$$

Participation constraints are satisfied if expected utility in the efficient equilibrium is not lower than the outside option: $u_i(x_1^*, x_2^*) \geq w_i$ for both $i = 1, 2$.[3] Simplifying these two inequalities

[3] We assume for simplicity that a consumer that contributes zero to the public good is excluded from consumption of the public good/the charity. An alternative assumption would be $u_i(x_1^*, x_2^*) \geq h_i(R - x_j^*) + w_i$. In this case additional calculations show that the condition on heterogeneity would be stricter ($b \in (0, 1/3] \cup [3, \infty)$) without leading to

using eq. (1) and (2) in combination with (9) and (10) leads to:

$$\frac{b(b+1)}{(b^2-\alpha)} \leq 2 \quad \text{and} \quad \frac{\alpha(b+1)}{(\alpha-b^2)} \leq 2. \tag{11}$$

Based on the heterogeneity measure b there are two cases to consider: Either, $b > 1$, in which case any efficient bias $\alpha \in (1, b^2)$ with the corresponding prize sum satisfies (6) and (7), or $b < 1$, in which case any efficient bias $\alpha \in (b^2, 1)$ satisfies (6) and (7), respectively. In the first case the left inequality in (11) can be satisfied by setting a feasible bias if $b > 2$, while the right inequality is always satisfied. In the second case the left inequality in (11) is always satisfied under any feasible bias, while the right inequality can be satisfied if $b < 1/2$. Hence, for a sufficient degree of heterogeneity, $b \in (0, 1/2) \cup (2, \infty)$, there exist feasible bias-prize combinations that lead to interior equilibria and induce the efficient amount of public good provision.[4] Naturally, this condition becomes stricter if the efficient amount $\hat{G}$ is induced with the minimal prize sum $R_{\min}$ (and corresponding bias $\alpha_{\max} = b$): Using the combination $(\alpha_{\max}, R_{\min})$ in (11) implies that interior and efficient equilibria exist if $b \in (0, 1/3] \cup [3, \infty)$.

For the general setup analyzed in section 3 the respective participation constraints of the two consumers can be reduced to the following inequalities: $\frac{\hat{h}^2}{(\hat{h}^2-\alpha)} \leq \frac{h_1(\hat{G})}{\hat{G}}$ and $\frac{\alpha}{(\alpha-\hat{h}^2)} \leq \frac{h_2(\hat{G})}{\hat{G}}$. Whether these inequalities hold simultaneously will depend on the complex interplay between functions h_1 and h_2, $\hat{G}$, $\hat{h}$, and bias α, which complicates the derivation of general results.

5 Concluding Remarks and Discussion

We analyze the introduction of biased lotteries, that can be distorted in favor of specific players, in a model of voluntary contribution to public good provision. Biasing the lottery in an appropriate way implies that competitive pressure among the players is increased which results in higher contributions to the public good. For the two player case we show that there exist bias-prize combinations that induce the efficient level of public good provision in any interior equilibrium of the respective voluntary contribution game with biased lotteries. Our approach therefore constitutes an improvement of Morgan (2000), where the use of unbiased lotteries leads to pareto-improvements in comparison to the pure voluntary contribution game without being able to achieve efficiency. At the same time our model preserves the advantages of the lottery

qualitatively different results.

[4] If $b \in [1/2, 2]$ the bias can be reduced (for a fixed prize sum) such that equilibria become interior. Numerical calculations show that this procedure will not result in the efficient amount $\hat{G}$ but still in higher public good provision than in the unbiased Morgan setup.

framework because its implementation does neither require coercive taxation nor own resources as the lottery prize is self-financed out of voluntary contributions.[5] This makes our approach specifically interesting in the context of charities and other non-public organizations.

While our formal setup is simple, it can also be applied to other environments. The voluntary contribution game with biased lotteries is, for instance, strategically equivalent to a voluntary contribution game with unbiased lotteries and price-discrimination, see Franke and Leininger (2014) for more details. Based on this alternative setup our results imply that it is possible to specify individualized prices for lottery tickets that induce the efficient level of public good provision in any interior equilibrium of the respective game. This optimal degree of price-discrimination is also closely related to the concept of Lindahl-pricing in the sense that lottery tickets are effectively (i.e. net of expected winnings from the lottery) priced at Lindahl-prices, see Franke and Leininger (2014). However, our approach also shares the typical limitations of Lindahl-pricing; that is, we assume complete information with respect to the preference parameters of the consumers because the optimal bias depends on the efficient level of public good provision which is by itself determined through individual preferences. Hence, our approach should not be interpreted as a novel method of preference elicitation.

Nevertheless, our results might also have implications in the case of a lottery organizer who is only partially informed about the underlying heterogeneity of the players: Based on numerical calculations we conjecture that favoring the weak player by biasing the lottery to some extent always increases public good provision and improves efficiency in comparison to unbiased lotteries, as long as the weak player is correctly identified and the bias is not exaggerated. Student rebates in charity lotteries would be a typical real world application suggested by our analysis.

A natural extension of our analysis is to consider a model with more than two players. While we conjecture that our results also hold in a multi-player game, the analysis is presumably more involved, for instance, because participation constraints have to be explicitly taken into account. Hence, the straightforward approach used in this paper might not be feasible in this more general setup. We plan to address this issue in future research.

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