Investments in managerial skills and bargaining over inputs. Is there hold-up?☆

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Abstract

This paper analyzes the incentives of the members of a committee to acquire skills, when they will share a fixed budget among them in ex-post negotiations. Skills are interpreted as the ability to manage a collective budget, in the sense that shares assigned to skilled agents generate positive externalities to all members. In this setting, the equilibrium generally displays an over-qualified population.

Key words: Investments, Hold-up, Multilateral bargaining, Skills, Externalities
JEL Classification: C78, J24, D62

1. Introduction

Consider a committee that must manage a budget to produce an output that will be shared among all its members according to some monotonically increasing default rule. Committee members negotiate how to distribute the budget among them after they have individually decided whether to acquire certain managing skills that will increase their productivity. Agents' preferences not only depend on the share of the output they finally receive but also on the share of the inputs they manage. So, there exists a conflict between particularistic and collective interests: inputs benefit those specific agents who manage them but, additionally, all agents benefit from the positive externalities generated by those inputs managed by skilled members because they increase the collective output level.[1]

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[1]A similar bargaining environment where agents face the conflict between particularistic and collective interests can be found in Volden and Wiseman (2007).

As a leading interpretation of the model, consider agents as the branch managers of a firm who must decide whether to increase their productivity and then negotiate over the distribution of inputs among their branches. Those managers participate on the profits/success of the firm but also benefit from the share of the budget they manage. More generally, the environment may be also interpreted as a multilateral bargaining game with positive consumption externalities, as in Calvert and Dietz (2005), where the size of such externalities depends on the endogenously selected capabilities/skills of the agents.

Most analysis of (irreversible) investment decisions on capabilities/skills in bargaining contexts assume that they directly affect the collective surplus, which is then shared in a bargaining game. In these settings, the distinction between specific and general investment becomes crucial to determine the (equilibrium) behavior of agents at the investment stage. It is widely known that when there is no market where agents can exploit their (acquired) abilities (*i.e.*, investments are specific) then the *hold-up problem* generates under-investments (see Grout (1984), Williamson (1985), Hart and Moore (1988), or a survey by Schmitz (2001)). As argued by some authors (e.g., Hart and Moore (1988), Che and Hausch (1999)) this is simply a consequence of incomplete contracts, which implies that agents cannot fully appropriate their marginal contributions in the ex-post negotiations. Instead, when investments decisions have some value in the market (*i.e.*, they are general), then ex-post negotiations do not necessarily induce to inefficient investments (see Chung (1991), Rogerson (1992), Aghion *et al.* (1994), Noldeke and Schmidt (1995), Edlin and Reichelstein (1996), Che and Sakovics (2004), and Evans (2008)).

Our starting point is different from the previous literature. In our model, skills affect the ability of the agents to manage the collective budget. Thus, higher abilities revert positively to the committee members (through a higher output) only when these skilled agents manage this budget. In other words, the available surplus of the committee at the bargaining stage does not depend exclusively on the skills of their members, but on how the budget is shared within the group.

We consider a committee with ex-ante identical impatient agents who face the decision of acquiring skills at some cost c or not. Then, a standard multilateral bargaining game will determine the allocation of a collective budget among them. In each stage of this dynamic bargaining game, one agent is chosen at random to propose a distribution of the budget whereas each of the remaining agents must respond to this proposal, either by accepting or rejecting it. The distribution is implemented only in case of unanimous acceptance; otherwise, the negotiation moves to the next period where the process re-starts. This budget will be used to produce a collective output. The productivity of unskilled agents is normalized to zero whereas skilled agents have a positive productivity. A typical agent's utility depends on both the size of the budget she manages and the collective output produced. So, she prefers the budget that is not managed by herself to be managed by a skilled rather than by an unskilled agent. For this reason, skills not only increase the (potential) productivity of the committee but also benefit the particular agent who acquires them, because she will be able to obtain a bigger share of the budget at the bargaining stage.

The relative size of the externalities determines the form of the unskilled agents' proposals at the negotiation stage. Specifically, we show that when externalities are low, equilibrium proposals assign a positive share of the budget to all responders. Hence, increasing the number of skilled agents also increases the total surplus. Moreover, this fact coexists with an additional efficiency problem, say the optimal number of skilled agents. Overall, we show that in equilibrium the number of agents acquiring skills exceeds what would be optimal. That is, an over-qualified population is generally obtained when agents endogenously select their skills. Efficiency is obtained only when either (i) the costs are sufficiently low and all agents acquire skills, or (ii) such costs are sufficiently high so that either no agent or only one agent becomes skilled.

For large externalities and sufficiently patient agents, the equilibrium proposals at the bargaining stage efficiently allocate the budget among skilled agents only. Hence, the optimal number of skilled agents is one when skills are costly. However, this is rarely obtained in equilibrium, because of the profitability of skills at the negotiation stage. The bargaining process is such that unskilled proposers gain the support of unskilled responders indirectly, through the positive externalities generated by the share of the budget assigned to skilled agents. Furthermore, when externalities are sufficiently large (some of) the skilled agents are over-compensated, in the sense that they obtain a share of the budget that is bigger than what they require to support the proposal. The number of those privileged agents depends on how unskilled proposers "coordinate", leading to multiple bargaining equilibria. To analyze the impact of coordination, we focus on the analysis of two extreme cases: the symmetric equilibrium, where the budget is equally split among all skilled agents; and the asymmetric equilibrium where only one skilled agent, say a preferential partner, is over-compensated. In the latter case, the incentives to acquire skills are smaller so the over-qualification problem is attenuated. Otherwise, unless the costs are sufficiently high, the private benefit from acquiring skills generally offsets the private costs and, consequently, over-qualification is obtained. On the other hand, under-qualification, which intuitively would be the reasonable consequence when considering a model of positive externalities, is obtained only when the investment costs are sufficiently high so that no agent acquires skills.

In the next section we present the model. In Section 3, we characterize the stationary subgame perfect equilibria of the game and compare the structure of the population obtained with that one maximizing collective surplus. Section 4 discusses some of the assumptions and extension of the model, and Section 5 concludes.

2. Model

A set of n agents, say N, interpreted as the branch managers of a firm play a two-stage game. First, they decide whether to invest $c > 0$ to increase their productivity/capability. Let $M \subseteq N$ represent the set of managers who invested c and let m be the number of managers in this set. Thus, the population is partitioned into two groups: one containing those m skilled managers, and another including $n - m$ unskilled managers. We denote by h and l a typical skilled and unskilled agent, respectively, and we let $c_h = c$, $c_l = 0$. Second, managers negotiate the division of a budget, normalized to one, among the different branches of the firm. This is done through a standard bargaining process that proceeds over discrete time as follows: at each period $t \geq 0$ one player $j \in N$ is randomly selected as the proposer, with equal probability each. This player proposes an allocation of the budget $\mathbf{x}^j = \left(x_1^j, ..., x_n^j\right) \in X = \left\{(x_1, ..., x_n) \in \mathbb{R}_+^n : \sum_{i=1}^n x_i \leq 1\right\}$, where x_i^j denotes the part of the budget assigned to player i. Then, sequentially all other players respond by either accepting the proposal or not. Unanimity is required for approval, so the budget is divided as prescribed by the proposal only if it receives the support of all responders. In case of approval, the proposed allocation is implemented and the game ends; otherwise the game moves to period $t + 1$ with a new randomly chosen proposer.

Managers' preferences are defined over both the particular share of the budget their branches receive and the total production of the firm. The productivity of unskilled managers is normalized to zero, whereas $\beta > 0$ denotes the productivity of skilled managers. Upon agreement on $\mathbf{x} \in X$ at period t player $j \in N$ obtains utility $\delta^t u_j(\mathbf{x}) - c_j$, where

$$u_j(\mathbf{x}) = x_j + \beta y,$$

$\delta \in (0, 1)$ and $y = \sum_{h \in M} x_h$. Perpetual disagreement yields utility $-c_j$.

A strategy for a given player should specify whether she invests c in the first stage and both a proposal and an acceptance/rejection rule for each subgame, in the second stage. The strategy is stationary if such proposal and acceptance rules are independent of the past history in the bargaining stage. A stationary subgame perfect equilibrium (henceforth, SSPE) is a profile of stationary strategies that are mutually best responses in each subgame.

An SSPE would determine a partition $(M, N \backslash M)$ of the population and a vector of expected utilities $\overline{u}_j(M)$ for any $j \in N$ such that: (i) $\overline{u}_j(M)$ is the expected utility obtained for agent j in an SSPE of the multilateral bargaining game where the population is partitioned into $(M, N \backslash M)$; and (ii) no player has incentives to change her investment decision. I.e.,

$$\begin{aligned} \overline{u}_j(M) &\geq \overline{u}_j(M \cup \{j\}) - c \text{ for all } j \in N \backslash M \\ \overline{u}_j(M) - c &\geq \overline{u}_j(M - \{j\}) \text{ for all } j \in M \end{aligned}$$

We will say that a partition satisfying the above inequalities is *stable*. Also, we will say that a partition $(M, N \backslash M)$ is *efficient* if [2]

$$W_m = \sum\nolimits_{j \in N} \overline{u}_j (M) - c \cdot m \geq \sum\nolimits_{j \in N} \overline{u}_j (M') - c \cdot m' = W_{m'} \text{ for any } M' \subseteq N.$$

3. Results

To characterize the stable partitions, we proceed by backwards induction: First, we characterize the equilibria of the bargaining game for any feasible partition of the population; and second, we analyze the stable partitions that arise when the involved agents anticipate the bargaining outcome in each subgame.

Negotiations proceed by agents submitting proposals to the committee. Unless $\beta > 1$, any proposer prefers the budget to be managed first by herself, second by skilled agents, and lastly by unskilled agents. Therefore, (optimal) proposals would assign to responders just the minimum share of the budget to gain unanimous consent.

The exact form of these proposals would depend on the relative size of externalities, β. In the Appendix it is shown that any SSPE is a no-delay equilibrium (Lemma 1). Moreover, it is proved that when $\beta(n - m) < 1$ then all responders receive exactly their expected continuation utility in the ensuing bargaining game (see Lemma 4). This implies that the bargaining equilibria must be symmetric, in the sense that all skilled (resp. unskilled) agents obtain the same expected utility at the bargaining stage (see Lemma 2). Instead, when $\beta(n - m) > 1$,[3] unskilled proposers give no share to unskilled responders (see Lemma 4), who are compensated indirectly by the positive externalities generated by the share of the budget assigned to skilled agents. Moreover, the skilled agents that are used by unskilled proposers as the channel to compensate unskilled responders receive more than their discounted expected utility; *i.e.*, they are *over-compensated*. The number of those privileged agents can range from 1 to m, leading to multiple equilibria when $m > 1$. In these cases, we will focus on the two extreme equilibria: (i) the symmetric equilibrium, where over-compensations are evenly split among all skilled agents; and (ii) the asymmetric equilibrium where only one skilled agent is over-compensated, say p, referred as the preferential agent. Regarding the proposals of skilled agents, Lemma 5 shows that when the equilibrium is symmetric and externalities are sufficiently large, $\beta > \delta/n$, then skilled proposers may obtain the support of unskilled responders by assigning them a zero share.[4] Contrarily, when $\beta < \delta/n$ the proposer

[2] Notice that this notion of efficiency is restricted. An absolute notion of efficiency would specify both m and the allocation of the surplus among agents that maximize aggregate utility, that would be selected independently of any bargaining process.

[3] To simplify the exposition we will exclude from the analysis the case $\beta (n - m) = 1$. Although m is endogenous, since it is a natural number, it can be generically excluded.

[4] Section B.2 in the Appendix, shows that this is always the case in the asymmetric equilibria.

should always put a positive share of the budget for unskilled responders in order to gain their support.

We next summarize the SSPE expected utilities for any possible partition. To simplify the exposition, we restrict our analysis to the outcomes that are attained as impatience vanishes, *i.e.* when $\delta \to 1$. This allows us to eliminate the effects of impatience on the results and focus the analysis only on the role of externalities in the ex-post negotiations. The calculations (for any δ) are provided in the Appendix. We will write $\overline{u}_j(M)$ as $\overline{u}_j$ when no confusion arises.

Proposition 1. *For any partition $(M, N \backslash M)$, as $\delta \to 1$ the symmetric equilibrium yields expected utilities*

$$\overline{u}_h = \begin{cases} \frac{\beta m+1}{m} & \beta > 1/n \\ \frac{\beta m+1}{(1-\beta(n-m))n} & \beta \leq 1/n \end{cases} \quad \textit{and } \overline{u}_l = \begin{cases} \beta & \beta > 1/n \textit{ and } m \geq 1 \\ 1/n & \textit{otherwise} \end{cases}$$

Proposition 2. *For any partition $(M, N \backslash M)$, as $\delta \to 1$ an asymmetric equilibrium exists if $m \in \{2, ..., n-2\}$ and $\beta > 1/(n-m)$. In these cases, $\overline{u}_l = \beta$ and*

$$\overline{u}_p = \begin{cases} \beta+1 & \beta > 1 \\ \beta+1-\frac{m-1}{n} & \beta \leq 1 \end{cases}, \ \overline{u}_h = \begin{cases} \beta & \beta > 1 \\ \beta+\frac{1}{n} & \beta \leq 1 \end{cases}$$

The previous propositions determine the agents' expected utilities at the bargaining stage. Importantly, we observe that the expected utility of unskilled agents is unaffected by the number of skilled agents, as far as $m \geq 1$. Moreover, skilled agents' expected utility decreases with m. This means that the acquisition of skills by one player generates negative effects that overcome its positive externalities.

Equipped with these results, we next focus on the first stage of the game (*i.e.*, when agents decide whether to acquire skills), to characterize the SSPE partitions and analyze their efficiency.

We first focus on the characterization of efficient partitions, referred as m^0.

Proposition 3. *For any $c > 0$ there is a unique efficient partition. If $\beta \leq \delta/n$ and $\delta \to 1$ then,*

1. *$m^0 = 0$ if $c > \frac{\beta}{1-\beta(n-1)}$*
2. *$m^0 \in \{1, 2, ..., n-1\}$ where m^0 satisfies*

$$c \in \left[\frac{\beta(1-n\beta)}{[1-\beta(n-m^0)]\,[1-\beta(n-m^0-1)]}, \frac{\beta(1-n\beta)}{[1-\beta(n-m^0)]\,[1-\beta(n-m^0+1)]} \right],$$

3. *$m^0 = n$ in case that $c < \frac{\beta(1-n\beta)}{1-\beta}$.*

If $\beta > \delta/n$ and $\delta \to 1$ then $m^0 = 1$ if $c < n\beta$ and $m^0 = 0$ otherwise.

Proof. When $\beta \leq \delta/n$ and $\delta \to 1$, the aggregate utility when there are m skilled agents is

$$W_m = \frac{1 + \beta(2m - n)}{1 - \beta(n - m)} - mc.$$

Then, it is easy to see that for any $m \in [0, n]$, $W_m > W_{m+1}$ if and only if

$$c > \frac{\beta\,(1 - n\beta)}{[1 - \beta(n - m)]\,[1 - \beta(n - m - 1)]}$$

and $W_m > W_{m-1}$ if and only if

$$c < \frac{\beta\,(1 - n\beta)}{[1 - \beta(n - m)]\,[1 - \beta(n - m + 1)]}.$$

On the other hand, when $\beta > \delta/n$ and $\delta \to 1$, the aggregate utility when $m > 0$ tends to

$$W_m = 1 + \beta n - mc.$$

Then, it is easy to see that for any positive c, $W_1 > W_m$ for any $m > 1$. On the other hand, $W_0 > W_1$ if and only if

$$1 > 1 + \beta n - c \Leftrightarrow c > \beta n.$$

■

For sufficiently patient agents, only skilled agents receive part of the budget when $\beta > 1/n$. Consequently, the gross aggregate utility remains constant for any $m \geq 1$. From this, it trivially follows that the efficient partition cannot contain more than one skilled agent. Contrarily, when $\beta < 1/n$ the proposer should always put a positive share of the budget for unskilled agents in order to gain their support. Thus, for moderate values of c and m, the gross aggregate utility grows with m because the lower is the number of unskilled agents the lower is the share of the budget for them and therefore the bigger are the positive externalities generated by skilled agents.

The characterization of the efficient partition turns out to be independent of the existence or not of multiple equilibria. However, expected utilities of skilled agents depend on how indirect compensations are made; *i.e.,* if they are evenly split among them or not. Clearly, the incentives for the acquisition of skills crucially depends on the bargaining equilibrium that we consider. We next study two extreme cases in turn: the symmetric bargaining equilibrium, and the asymmetric bargaining equilibrium where only one skilled agent (the preferential one) is over-compensated.

3.1. Symmetric bargaining equilibrium

In a symmetric equilibrium all players with the same skills obtain the same expected utility. Let m^* denote the number of skilled agents in an SSPE. Next result, characterizes the SSPE for low investment returns; *i.e.,* a low β.

Proposition 4. *Let $\beta \leq \delta/n$ and $\delta \to 1$. Then, for any $c > 0$ there is a unique symmetric SSPE yielding*

1. $m^* = 0$ *if* $c > \frac{\beta}{1-\beta(n-1)}$
2. $m^* \in \{1, 2, ..., n-1\}$ *where* m^* *satisfies*

$$c \in \left[\frac{\beta}{1-\beta(n-m-1)}, \frac{\beta}{1-\beta(n-m)}\right].$$

3. $m^* = n$ *in case that* $c < \beta$.

Proof. We compare the equilibrium expected utilities given in Proposition 1 to analyze the stability of every possible partition. First, $m^* = n$ if no skilled agent prefers to be unskilled; *i.e.,* when

$$\frac{1}{n} - \frac{\beta n + 1}{n} + c \leq 0 \Rightarrow c \leq \beta.$$

Similarly, $m \in \{1, n-1\}$ can be sustained in equilibrium if (i) no skilled agent prefers to be unskilled; *i.e.,*

$$\frac{1}{n} - \frac{\beta m + 1}{(1-\beta(n-m))n} + c \leq 0 \Rightarrow c \leq \frac{\beta}{1-\beta(n-m)}$$

and (ii) no unskilled agent prefers to become skilled, *i.e.,*

$$\frac{\beta(m+1)+1}{(1-\beta(n-m-1))n} - \frac{1}{n} - c \leq 0 \Rightarrow c \geq \frac{\beta}{1-\beta(n-m-1)}.$$

Finally, $m^* = 0$ if no unskilled has incentives to acquire skills; *i.e.,* when

$$\frac{\beta+1}{(1-\beta(n-1))n} - \frac{1}{n} - c \leq 0 \Rightarrow c \geq \frac{\beta}{1-\beta(n-1)}.$$

■

By investing c, agents generate externalities to others, which allow them to obtain a larger share when proposing in the ensuing negotiations. Additionally, because of unanimity and symmetry, this also implies that other proposers must give them a larger share in order to gain their support. Thus, as far as the profitability of these two effects overcome the investment costs (which happens for relatively low values of m and c), unskilled agents will have incentives to acquire skills. As shown next, in most cases, this generates an over-qualified population.

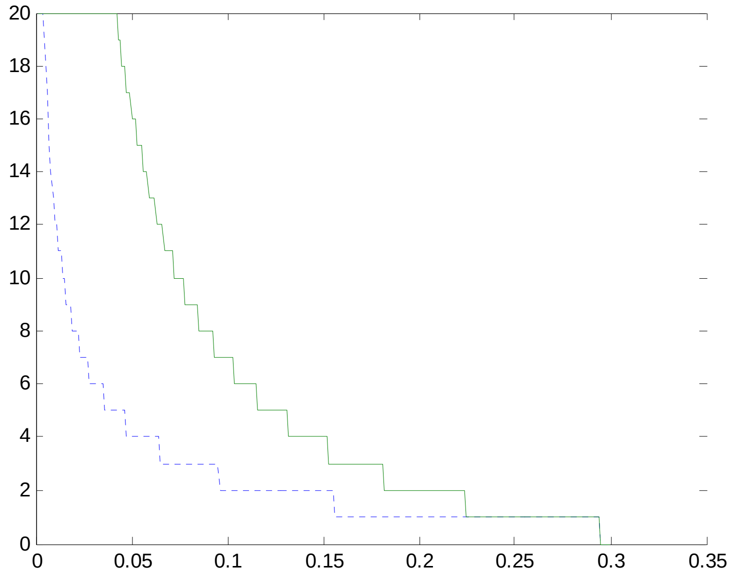

Figure 1: The SSPE (solid line) and the efficient (dashed line) number of skilled agents when $n = 20$ and $\beta = 0.045$.

Corollary 1. *Let $c > 0$, $\beta \leq \delta/n$ and $\delta \to 1$. The SSPE displays over-investment when $c \in \left(\frac{\beta(1-n\beta)}{1-\beta}, \frac{\beta}{1+2\beta-\beta n}\right)$. Otherwise, the SSPE is efficient.*

Proof. Immediate from Propositions 3 and 4. ■

At first glance, this can be seen as a counter-intuitive result in this setting with positive externalities. However, the entry of a new high productivity agent generates a negative effect on the rest of skilled agents (if any) because they will have to increase the share to the deviator in order to gain her support. This erodes the effects of positive externalities of the new skilled agents and causes the over-qualification for low costs, where the SSPE yields $m^* \geq 2$. Figure 1 illustrates the comparison between efficient and SSPE partitions for any c when $\beta \leq 1/n$.

Next, we turn to the case with high investment returns.

Proposition 5. *Let $\beta > \delta/n$ and $\delta \to 1$. Then, there is a unique symmetric SSPE yielding*

1. $m^* = 0$ *if* $c > \frac{n(1+\beta)-1}{n}$
2. $m^* = 1$ *whenever* $c \in \left[\frac{1}{2}, \frac{n(1+\beta)-1}{n}\right]$
3. $m^* \in \{2, ..., n-1\}$ *where* m^* *satisfies* $c \in \left[\frac{1}{m+1}, \frac{1}{m}\right]$.

4. $m^* = n$ *if* $c \leq \frac{1}{n}$.

Proof. Similar to the proof of Proposition 4, thus omitted. ■

The private benefits of being skilled in the negotiations when $\beta \leq \delta/n$ also apply when $\beta > \delta/n$. Moreover, another effect may appear in these cases: when unskilled agents are compensated indirectly, then skilled agents are over-compensated, which increases the profitability of acquiring skills. Furthermore, as agents become infinitely patient ($\delta \to 1$), the equilibrium share obtained by skilled agents tends to 1.[5] Hence, by symmetry, each skilled agent obtains $1/m$ of the budget, which reinforces the profitability of skill investments.

A corollary similar to Corollary 1 is obtained.

Corollary 2. *Let* $c > 0$, $\beta > \delta/n$ *and* $\delta \to 1$. *Then, the SSPE is efficient if either* $c \in \left(\frac{1}{2}, \frac{n(1+\beta)-1}{n}\right)$ *or* $c > n\beta$. *If* $c < 1/2$ *then there is over-investment and if* $c \in \left(\frac{n(1+\beta)-1}{n}, n\beta\right)$ *under-investment is obtained.*

Figure 2 illustrates this comparison between efficient and SSPE partitions for any c when $\beta > \delta/n$ and $\delta \to 1$. As in the case of small externalities, the equilibrium displays over-investment whenever $m^* \geq 2$. This is due to the negative effect that the entry of an additional skilled agent generates on the other skilled. However, this inefficiency is more pronounced when $\beta > \delta/n$, because the efficient partition contains only a single skilled agent whenever $c < n\beta$. A second remarkable difference with respect to the previous case is that now there is under-investment when $c \in \left(\frac{n(1+\beta)-1}{n}, n\beta\right)$. In this interval, $m^* = 0 > m^0 = 1$. Hence, the acquisition of skills by an agent when there are no skilled agents in the committee cannot cause any negative effect, as described above. *I.e.,* all agents would benefit from the presence of one skilled agent. However, the utility gain of a unique agent investing c is lower than the aggregate utility gain and this leads to under-qualification when $c \in \left(\frac{n(1+\beta)-1}{n}, n\beta\right)$. This contrasts with the case when $\beta < \delta/n$ and $\delta \to 1$, where there is no under-investment because the unique skilled agent can absorb the aggregate gains of her investment.

3.2. Preferential partner

Next, we analyze the stability of partitions when there is a preferential skilled agent, in the sense that only one skilled agent is (possibly) over-compensated at the bargaining stage. The rest of skilled agents are called ordinary skilled agents. In this case, two equilibria may be attained. In one of them, there is only one skilled agent

[5] In Section 4.1, we discuss the effects of considering impatient agents.

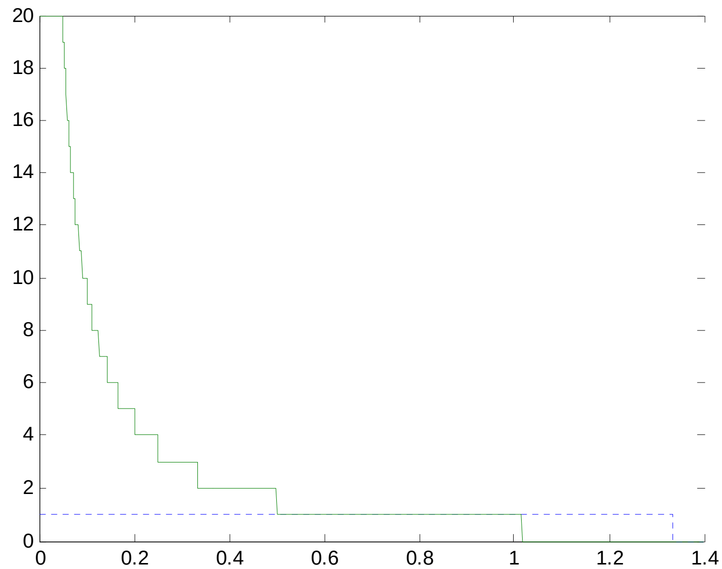

Figure 2: The SSPE (solid) and the efficient (dashed) number of skilled agents when $n = 20$ and $\beta = 1/15$.

because others are deterred from acquiring skills by the threat of an asymmetric bargaining outcome. Despite this, when the number of skilled agents is sufficiently high the bargaining outcome must be symmetric and, therefore, there may also exist a partition with $m^* > 1$ which is stable. Next proposition summarizes these claims.

Proposition 6. *Let $c > 0$ and assume there is a preferential partner. If $\beta < 1/(n-2)$ or $n = 2$ then the SSPE is given by Propositions 4 and 5. Otherwise, when $\beta \geq 1/(n-2)$, the SSPE yields:*

1. *If $c \leq 1/n$ then*
   (a) *$m^* = n$ and*
   (b) *$m^* = 1$ only if $\beta > 1$.*
2. *If $c = 1/n$ then $m^* \in \{2, ..., n-3\}$, where $\beta \in \left(\frac{1}{n-m^*-1}, 1\right]$.*
3. *If $c \in (1/n, \beta/(\beta n - 1))$ then there are two equilibria for the same c,*
   (a) *$m^* = 1$ and*
   (b) *$m^* \in \{2, ..., n-1\}$ when $c \in \left[\frac{1}{m^*+1}, \frac{1}{m^*}\right]$.*
4. *If $c > \beta/(\beta n - 1)$ then $m^* = 1$ if $c \leq \frac{n(1+\beta)-1}{n}$ and $m^* = 0$ otherwise. In addition, when $c \leq 1/(n-1)$ and $\beta > 1$ then $m^* = n-1$ is also stable.*

The proof of this proposition can be found in the Appendix. The existence of an asymmetric bargaining outcome depends on how many agents acquire skills. In particular, if m is such that $\beta(n-m) < 1$, the bargaining equilibrium must be symmetric. In contrast, when only a few agents acquire skills so that $\beta(n-m) > 1$, then the bargaining equilibrium exhibits a strong asymmetry among skilled agents. The reason is that only the preferential player p is over-compensated by unskilled proposers to gain the support of unskilled responders. Furthermore, this extreme coordination alleviates the inefficiency problem discussed above as it dissuades unskilled from investing c. It is worth to note that, an asymmetric SSPE is obtained only in the non-generic case where $c = 1/n$. Otherwise, when $c \neq 1/n$, the threat of an asymmetric bargaining outcome deters agents to acquire skills and therefore no asymmetric bargaining outcome is never obtained in equilibrium.

Corollary 3. *Consider that there is a preferential agent. If $\beta > 1$ there is always an efficient SSPE except when $c \in \left(\frac{n(1+\beta)-1}{n}, n\beta\right)$, where there is under-investment. If $\beta \in [1/(n-2), 1]$ there are two regions without any efficient SSPE: $c < 1/n$ (over-investment) and $c \in \left(\frac{n(1+\beta)-1}{n}, n\beta\right)$ (under-investment). If $\beta < 1/(n-2)$ then results do not change with respect to the symmetric case.*

As an illustration, take the example depicted in Figure 2 where $\beta = 1/15$ and $n = 20$, and consider that there is a preferential partner. In Figure 3, SSPE stable partitions are represented by the solid line, and the dashed line refers to the efficient partition. Despite the fact that only one agent would be over-compensated in the negotiations, when $c < 1/n$ only the symmetric equilibrium exists. Moreover, when $c \in (1/n, \beta/(\beta n - 1))$, the existence of a preferential player induces two SSPE. On the one hand, if $m^* > 5$ then $\beta < 1/(n-m^*)$ and therefore the bargaining equilibrium must be symmetric. On the other hand, if $m^* = 1$ then $\beta > 1/(n-m^*)$ and the threat of an asymmetric bargaining outcome deters the acquisition of skills by other agents. In case that $c \in [\beta/(\beta n - 1), (n(1+\beta)-1)/n]$, any symmetric SSPE would be such that $m < 5$ (*i.e.*, $\beta > 1/(n-m)$) and this is unstable in case of existing a preferential agent. Hence, $m^* = 1$ is the unique stable partition. When $c > (n(1+\beta)-1)/n$ then no agent can compensate the entry costs and therefore $m^* = 0$. Thus, there is under-investment when $c \in ((n(1+\beta)-1)/n, n\beta)$. Hence, there exists an efficient SSPE when $c \in (1/n, (n(1+\beta)-1)/n)$.

As a final remark, note that in between the homogeneous distribution of indirect compensations (symmetric case) and the presence of a preferential agent, there are many intermediate cases where coordination is not so sharp. For instance, unskilled might use a subset of (preferential) skilled to indirectly compensate other unskilled; or possibly only a fraction of unskilled uses the same skilled agent whereas others split their indirect compensations. In these intermediate cases, skilled agents will have equilibrium expected utilities that will range from the utility of the ordinary skilled agents in the preferential partner case to the expected utility of skilled agents in the symmetric case. This will reduce the situations with over-qualification levels that will be in between these two extremes.

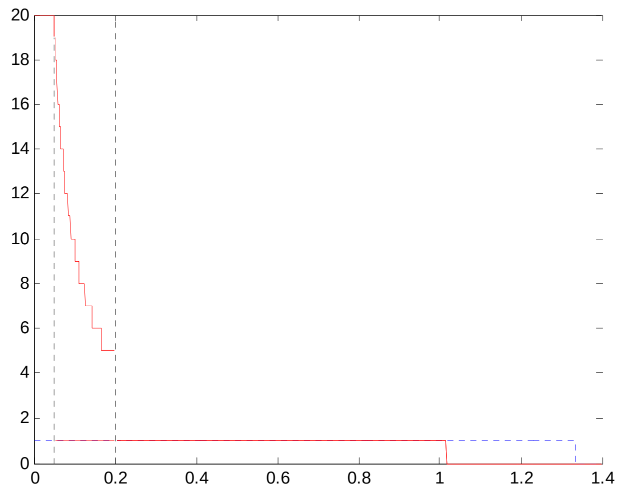

Figure 3: The SSPE (solid) and the efficient (dashed) number of skilled agents when $n = 20$, $\beta = 1/15$ and there is a preferential player.

4. Discussion

4.1. Strictly impatient players

By analyzing the limit case $\delta \to 1$, we omitted the role of impatience in shaping the equilibrium proposals and (expected) utilities at the bargaining stage. We next detail the effects of considering lower values of δ. First, notice that a lower δ increases the costs of delaying the agreement, so responders would be willing to accept less favorable agreements. Consequently, the lower δ the lower the share received by responders.[6] Hence, δ will obviously affect the equilibrium utilities, as it can be deduced from the general expressions in the Appendix.

In the limit case $\delta \to 1$, the equilibrium proposals can be classified into two different categories: when $\beta < 1/n$, proposers need to give a positive share of the budget to all responders in order to gain their approval, whereas unskilled responders receive no share when $\beta > 1/n$. However, for a general sufficiently high δ, three cases are distinguished: First, when $\beta < \delta/n$, all responders should receive a positive share of the budget for sufficiently high values of δ. Second, when $\beta \in (\delta/n, 1/(n-m)]$ skilled proposers do not need to put any share of the budget for unskilled agents for any δ whereas unskilled proposers have to give a positive share to unskilled responders to gain their approval. This share tends to zero as impatience vanishes, so these equilibrium proposals cannot be distinguished from the case discussed next

[6] For instance, in the limit when $\delta \to 0$, proposers are able to retain the whole budget for themselves.

when we consider $\delta \to 1$. Finally, when $\beta > 1/(n-m)$ unskilled responders receive no share of the budget from either skilled or unskilled proposers. The reason is that (i) skilled proposers do not need to give a positive share to unskilled responders to gain their approval (Lemma 5) and (ii) unskilled proposers prefer to gain the support of the rest of unskilled agents by exploiting the positive externalities generated by the share of the budget managed by skilled agents (Lemma 4). This obviously benefits those skilled agents that are used as a channel to gain the support of unskilled responders. Moreover, these indirect compensations make possible the existence of asymmetries among skilled agents in the negotiations, as in the preferential player case.

4.2. Other consensus requirements

A second simplifying assumption of our baseline model is the requirement of unanimity in the negotiation stage. Under unanimity, acquiring skills is particularly profitable because the increase in the expected utility obtained by skilled agents when proposing positively affects the utility they obtain when responding. Contrarily, under weaker consensus requirements, proposers can exclude skilled agents from the winning coalition if their demands are relatively high. Thus, some potential benefits from acquiring skills are eroded. In this respect, the size of β will also play a key role in determining whether (more demanding) skilled agents will be included in any winning coalition.[7] In any case, lower consensus requirements would reduce the incentives to acquire skills and, consequently, would alleviate the over-investment problem detected in our baseline model. In equilibrium, proposers would use mixed strategies that exclude skilled agents from the winning coalition with certain probability in order to equalize the net benefits of including skilled and unskilled agents in the winning coalition. Further research could aim at formalizing this analysis.

4.3. Stability and efficiency

The results of the previous section offer a clear image of the conflict between efficiency and stability. The intuition behind this conflict is detailed next.

When the acquisition of skills is sufficiently costly, the efficiency of the equilibrium outcome depends on β; *i.e.,* the size of the positive externalities generated by skilled agents. When these externalities are moderate ($\beta < \delta/n$), the presence of one skilled agent does not affect unskilled equilibrium utilities because the skilled agent can absorb the whole surplus she generates. Consequently, the skilled agent's marginal utility for her investment coincides with the aggregate marginal utility.

[7] These claims are in accordance to Eraslan (2002), who highlights that being stronger in unanimity bargaining games does not necessarily apply when consensus requirements are weaker than unanimity.

This precludes the possibility of under-investments and therefore, the resulting partition is efficient. Contrarily, if the productivity gains from acquiring skills are high ($\beta > \delta/n$), the agent who invests c cannot absorb the whole surplus generated by her investment. Although she retains all the productive resources for herself, the rest of managers benefit from the high positive externalities she generates, since their expected utility of unskilled changes from $1/n$ to β. However, since the investor does not internalize these gains, there is under-investment, as in a typical problem of positive externalities.

When acquiring skills is not so costly, then the bargaining process among managers generates negative externalities that cause over-investment: if an additional manager acquires skills then all other skilled managers must transfer part of their share of productive resources to her in order to gain her support, so their expected utility in the equilibrium of the bargaining game decreases. In this respect, discouraging the acquisition of skills will alleviate over-investment. This may happen when $\beta(n-m) > 1$ and only a subgroup of skilled managers are used by unskilled proposers as a channel to gain the support of the rest of unskilled agents. In the extreme case where this subgroup contains only one agent, the threat of asymmetric treatment of skilled agents supports an SSPE where the over-investment inefficiency problem is (almost) solved.

4.4. The interpretation of skills

One could think that over-skill-investment arises in equilibrium because skills benefit not only the rest of agents but also the investor. Remember that the latter obtains a fixed part of the final output, so that she indirectly benefits from having a higher productivity whenever she manages a positive share of productive resources. Alternatively, one could conceive a setting without this extra benefit from investment; *i.e.,* where the utility function of an agent j is

$$u_j(\mathbf{x}) = x_j + \beta y,$$

where $y = \sum_{h \in M \setminus \{j\}} x_h$. In this case, the share of the budget that obtains an agent who invested c generates positive externalities to the rest but it does not contribute to indirectly increase her own utility. As an example, we can think on a set of agents that negotiate how to distribute a divisible consumption good but, beforehand, they had the opportunity to invest c to endear themselves to the rest of agents: The share of the consumption good of an agent who invested c generates positive consumption externalities to the rest.

By considering this alternative version of the model, results do not change qualitatively:[8] Both the equilibrium proposals at the bargaining stage and the equilibrium/efficient partitions remain essentially the same. There is over-investment

[8] Available upon request.

for c sufficiently low, and under-investment for a sufficiently high c when β is also sufficiently high, as in the model considered in the paper. Consequently, it should be concluded that the fundamental motivation to acquire skills in our model has not to do with the extra benefit that skilled agents obtain, but with the role of the externalities at the negotiation stage.

5. Conclusions

In this paper we analyzed the acquisition of skills by the branch managers of a firm in a setting where (i) they do not only benefit from the success of the firm but also from the share of inputs assigned to their branches, and (ii) managers distribute inputs among branches following a standard non-cooperative bargaining process that requires unanimous consent. Productive resources managed by skilled agents generate positive externalities to the rest because they contribute to increase firm's production.

We characterized the equilibrium proposals and utilities of the participants of the bargaining stage for any partition of the population into skilled and unskilled managers, and for any discount rate. Then, we focused on arbitrarily patient agents to show how their proposals are affected by the size of externalities. In particular, when externalities are low a proposal is unanimously accepted if and only if all agents receive a positive share of inputs, whereas skilled agents will manage all inputs if externalities are high. Moreover, for sufficiently large externalities, we showed that there are multiple bargaining equilibria, where the budget is asymmetrically shared among skilled agents.

In this framework with positive externalities, one could reasonably expect to find under-investment on productivity skills. However, the results generally confirm just the opposite. The reason has to do with the profitability of skills in the ex-post negotiations. In particular, unless the costs of acquiring skills are high and there is only one skilled agent, the negotiation process involves that skills generate negative externalities: when an additional manager invests on productivity skills then all other skilled managers must transfer part of their share of productive resources to her in order to gain her support and reach unanimous consent. This decreases the utility of skilled managers. These negative effects lead to over-investment. In this respect, the presence of a preferential partner (or a small set of preferential partners) allows to concentrate productive resources into few skilled managers, which alleviates the over-investment.

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A. Preliminaries

Lemma 1. *Any SSPE is a no-delay equilibrium and* $\sum_{i\in N} x_i^j = 1$ *for all* $j \in N$.

Proof. Consider an SSPE yielding expected utilities $(\overline{u}_1, ..., \overline{u}_n)$.

As agents are risk neutral, $A = \left\{\mathbf{x} \in \mathbb{R}_+^n, \sum x_i^j \leq 1 : u_i(\mathbf{x}) \geq \overline{u}_l \text{ for all } i \in N\right\} \neq \varnothing$. Hence, for any $\delta < 1$,

$$A(\delta) = \left\{\mathbf{x} \in \mathbb{R}_+^n, \sum x_i^j < 1 : u_i(\mathbf{x}) \geq \delta\overline{u}_l \text{ for all } i \in N\right\} \neq \varnothing.$$

Therefore, the proposer j can obtain more than $\delta\overline{u}_j$ when proposing, implying that there is no delay. Moreover, j will exhaust all resources when proposing so that $\sum_{i\in N} x_i^j = 1$. ■

Lemma 2. *Any SSPE where identical responders obtain exactly their expected utility or a zero share yields the same expected utility for identical players.*

Proof. Consider an equilibrium yielding $\overline{u}_1 \leq ... \leq \overline{u}_m$ and $\overline{u}_{m+1} \leq ... \leq \overline{u}_n$, where $1, .., m$ are skilled agents. We proof the statement for $h - type$ agents. A similar argument works for $l - type$ agents.

Let $x^m = (x_m^m, x_1^m, H, L)$ be the optimal proposal of agent m, where $H = \sum_{h\notin\{1,m\}} x_h^m$ and $L = \sum_{l\in N\setminus M} x_l^m$. This yields

$$u_m(x^m) = (1 - x_1^m - H - L) + \beta(1 - L), \tag{1}$$
$$u_1(x^m) = x_1^m + \beta(1 - L). \tag{2}$$

Agent 1 can induce the acceptance of $y^b = (y_1^1, y_m^1, H, L)$, with $u_m(y^1) = \delta\overline{u}_a$ if $y_m^1 > 0$ and $y_m^1 = 0$ otherwise. This would yield,

$$u_m\left(y^1\right) = y_m^1 + \beta(1 - L), \tag{3}$$
$$u_b\left(y^b\right) = \left(1 - y_m^1 - H - L\right) + \beta(1 - L). \tag{4}$$

Using (1) and (4), we obtain

$$u_m(x^m) - u_1\left(y^1\right) = y_m^1 - x_1^m,$$

and from (2) and (3),

$$u_m\left(y^1\right) - u_1(x^m) = y_m^1 - x_1^m.$$

Hence,

$$u_m\left(y^1\right) - u_1(x^m) = u_m(x^m) - u_1\left(y^1\right). \tag{6}$$

Let x^1 be an optimal proposal of agent 1. Note that

$$x_m^i > 0 \text{ for some } i \notin \{1, m\} \text{ implies } u_1^i \geq \delta \overline{u}_1 \text{ and } u_m^i = \delta \overline{u}_m, \tag{7}$$

and

$$x_m^i = 0 \text{ for some } i \notin \{l, h\} \text{ implies } u_1^i \geq u_m^i. \tag{8}$$

Moreover,

$$\begin{aligned}(n+1)\,\overline{u}_m &= \sum_{i \notin \{1,m\}} u_m^i + u_m\left(x^1\right) + u_m\left(x^m\right) \text{ and} \\ (n+1)\,\overline{u}_1 &= \sum_{i \notin \{1,m\}} u_1^i + u_1\left(x^1\right) + u_1\left(x^m\right).\end{aligned}$$

Therefore,

$$\left[(n+1) - n_1\delta\right]\left[\overline{u}_m - \overline{u}_1\right] \leq \left[u_m\left(x^m\right) - u_1\left(x^1\right)\right] + \left[u_m\left(x^1\right) - u_1\left(x^m\right)\right], \tag{9}$$

where $n_1 \leq n-1$ is the number of players proposing $x_m^i > 0$.

We next distinguish two cases: either $y_m^1 = 0$ or $y_m^1 > 0$.

Case 1: $y_m^1 = 0$ implies that the optimal proposal of agent 1 satisfies $x^1 = y^1$, since x^m is optimal for m. Thus, $u_m\left(x^m\right) = u_1\left(x^1\right)$ and $u_m\left(x^1\right) = u_1\left(x^m\right)$, which (using (9)) contradicts $\overline{u}_m > \overline{u}_1$.

Case 2: $y_m^1 > 0$. In this case, the optimal proposal of agent 1 is such that either $x_m^1 > 0$ or $x_m^1 = 0$.

Case 2.1: Note that $x_m^1 > 0$ implies $u_m\left(x^1\right) = \delta \overline{u}_m = u_m\left(y^1\right)$. Moreover, $u_1\left(x^1\right) \geq u_1\left(y^1\right) \geq \delta \overline{u}_b$. Thus, using (6), we obtain

$$u_m\left(x^m\right) - u_1\left(y^1\right) = u_m\left(y^1\right) - u_1\left(x^m\right) = u_m\left(x^1\right) - u_1\left(x^m\right) \leq \delta\left[\overline{u}_m - \overline{u}_1\right].$$

Hence, using (9) and $\overline{u}_a > \overline{u}_b$ we get a contradiction.

Case 2.2: If $x_m^1 = 0$ then it must be that $x_1^m = 0$, too. This implies that $x^1 = y^1$ and therefore, as in Case 1, a contradiction is obtained. ■

Next, we show some preliminary results that will be used below. First, we focus on symmetric equilibria, where all players with the same skills have the same expected utilities.

Lemma 3. *In any symmetric SSPE*

$$\overline{u}_l = \frac{1}{n-m} + \frac{m\left(\beta - \frac{1}{n-m}\right)}{1+\beta m}\overline{u}_h. \tag{10}$$

Proof. Because of the symmetry of SSPE and the linearity of the utility functions, any SSPE can be represented by a pair of expected shares $(\overline{x}_h, \overline{x}_l)$ such that $m\overline{x}_h = 1 - (n-m)\overline{x}_l$ for some $\overline{x}_l \in [0, 1/(n-m)]$. These expected shares yield:

$$\overline{u}_h = (1+\beta)\overline{x}_h + (m-1)\beta\overline{x}_h = \overline{x}_h(1+\beta m),$$

and

$$\overline{u}_l = \overline{x}_l + \beta m\overline{x}_h = \frac{1}{n-m}(1 - m\overline{x}_h) + \beta m\overline{x}_h = \frac{1}{n-m} + \overline{x}_h\left(\beta m - \frac{m}{n-m}\right).$$

Thus,

$$\overline{u}_l = \frac{1}{n-m} + \frac{m(\beta - \frac{1}{n-m})}{1+\beta m}\overline{u}_h.$$

■

Notice that, if $\beta > 1$ then unskilled proposers will prefer to give the whole budget to skilled agents. In all other cases, proposers prefer the budget to be managed first by themselves, second by skilled agents, and lastly by unskilled agents. The requirement of unanimous consent may force proposers to allocate shares of the budget to the least preferred destinations. The following two preliminary lemmas explore these underlying forces to characterize agents' proposals.

Lemma 4. *If $\beta > 1/(n-m)$ and $m \geq 1$ then unskilled proposers will give no share of the budget to unskilled responders.*

Proof. Note first that, since unskilled agents receive at mot their expected utility when responding, by Lemma 2 all unskilled agents have the same expected utility in any SSPE.

The claim is immediate if $m = n-1$. When $m \in \{1, ..., n-2\}$ the proposal of an unskilled player i can be written as $\mathbf{x}_i = (1-y-x, y, x)$ where $1-y-x \geq 0$ is what the proposer keeps for herself, $x \geq 0$ represents the total proposed share for unskilled responders, and $y \geq 0$ is the proposed share for all skilled players. Player i has two different alternatives to gain the support of other unskilled agents: (a) giving a sufficient y to indirectly convince unskilled agents through the externalities generated by skilled agents (indirect compensation) or (b) giving a sufficient x_l^i (direct compensation) to each unskilled responder l. We claim that alternative (a) is preferred to (b) if and only if $\beta > 1/(n-m)$. To prove this claim, notice that $\mathbf{x}_i$ generates utilities:

$$\begin{aligned} u_i &= 1 - y - x + \beta y \text{ and} \\ u_l &= x_l^i + y\beta, \end{aligned}$$

where u_l represents the utility of a generic unskilled non-proposer. Agent i may increase u_l for all $l \in N \backslash M - \{i\}$ using alternatives (a) or (b) given below:

(a) $dy > 0$ and $dx = 0$. This implies $dy = \frac{1}{\beta}du_l$ and $du_i^{(a)} = -\frac{1-\beta}{\beta}du_l$.

(b) $dy = 0$ and $dx = (n-m-1)\, dx_l^i > 0$. This implies $dx = du_l$ and $du_i^{(b)} = -(n-m-1)\, du_l$.

Thus, player i prefers alternative (a) iff $du_i^{(a)} > du_i^{(b)}$; *i.e.*, iff $\beta > \frac{1}{n-m}$. ■

Corollary 4. *In any SSPE, if $\beta < 1/(n-m)$ then all responders receive either a zero share or their discounted expected utility.*

Proof. From the proof of previous Lemma, it is immediate that when $\beta < \frac{1}{n-m}$, skilled agents are never used as the channel to compensate unskilled responders. Thus, they will never receive more than their expected utility when responding. Moreover, as any unskilled responder generates less externalities (zero) than any other player, these agents can never be over-compensated and receive a positive share, simultaneously . ■

Notice that the previous Lemma does not require that unskilled proposers follow any particular pattern to distribute the budget among skilled agents. Nevertheless, our analysis focuses on two extreme distributions: (1) unskilled proposers give the same share of the budget to every skilled responder (symmetric case) and (2) unskilled proposers concentrate all indirect compensations through a unique skilled agent (asymmetric case). Obviously, between these two extremes there are many intermediate cases, whose analysis is omitted here. The following lemma applies for the symmetric case.

Lemma 5. *In the symmetric case, skilled proposers will give no share of the budget to unskilled responders when $\beta > \delta/n$.*

Proof. The claim is obvious when $m \in \{0, n\}$. Assume $m \in \{1, ..., n-1\}$. A skilled proposer can offer no share of the budget to unskilled agents if their utility from accepting the current proposal, β, is higher or equal than their expected utility from having a new proposer selected following a delay, $\delta\overline{u}_l$. Let us find $\overline{u}_l$. In the symmetric case, an unskilled proposer i sets either $\mathbf{x}_i = (1-my-(n-m-1)x, y, x)$ when $m \in \{1, ..., n-2\}$, where $1-y-(n-m-1)x \geq 0$ and $x \geq 0$ ($y \geq 0$) represents the proposed share for every unskilled (skilled) responder or $\mathbf{x}_i = (1-mz, z)$ when $m = n-1$, where $1-mz \in [0,1]$ and z represents the proposed share of the budget for any skilled agent. In order to maximize her utility, player i sets x, y, and z such that:

$$\begin{aligned} x + my\beta &= \delta\overline{u}_l, \\ y(1+\beta) + (m-1)y\beta &= \delta\overline{u}_h, \text{ and} \\ z(1+\beta) + (m-1)z\beta &= \delta\overline{u}_h. \end{aligned}$$

The expected utility of an unskilled agent can be written as:

$$\overline{u}_l = \begin{cases} \frac{1}{n}(1 - my - (n-m-1)x + my\beta + m\beta + (n-m-1)\delta\overline{u}_l) & \text{if } m \in \{1, ..., n-2\} \\ \frac{1}{n}(1 - mz + mz\beta + m\beta) & \text{if } m = n-1 \end{cases},$$

whereas Equation (10) can be used to obtain $\overline{u}_h$. By solving this system of equations we get

$$\overline{u}_l = \frac{1 - \delta + \beta m}{n - \delta(n-m)}$$

for any $m \in \{1, ..., n-1\}$. Thus, $\beta \geq \delta\overline{u}_l$ if and only if $\beta \geq \delta/n$. ■

B. Bargaining Equilibrium

In what follows we calculate the equilibrium allocations and utilities for any possible partition $(M, N\backslash M)$. We first consider symmetric bargaining equilibria.

B.1. Symmetric case

The analysis is divided into the following three subcases:

B.1.1. $m = n$

In this case, the proposer keeps $1 - (n-1)x_1$ for herself and puts x_1 for each other player, where $x_1 \in [0, 1/(n-1)]$. At the voting stage, responders weigh their utility from the current proposal, $x_1 + \beta$, against the expected utility from having a new proposer selected following a delay, which by symmetry equals $\frac{\delta(\beta n+1)}{n}$. Thus, non-proposers accept the proposal iff

$$x_1 + \beta \geq \frac{\delta(\beta n + 1)}{n} \Leftrightarrow x_1 \geq \frac{\delta - \beta n(1-\delta)}{n}.$$

To maximize her utility, the proposer sets $x_1 = \max\left\{0, \dfrac{\delta - \beta n(1-\delta)}{n}\right\}$. This proposal is accepted by all players. Thus, their expected utility is $\overline{u}_h = \dfrac{\beta n + 1}{n}$.

B.1.2. $m \in [1, n-1]$

Equilibrium proposal will depend on β as follows:

Case 1: $\beta > 1$. Trivially, in the symmetric case, unskilled proposers give $1/m$ to each skilled agent whereas, by Lemma 5, any skilled proposer keeps $1-(m-1)x_2$ for herself and puts x_2 for every skilled responder (if $m > 1$), where $x_2 \in [0, 1/(m-1)]$. This proposal is accepted by skilled responders if and only if their utility from the current proposal, $x_2 + \beta$, is higher or equal than the expected utility from having a new proposer selected following a delay, $\delta(\beta m + 1)/m$; *i.e.*,

$$x_2 + \beta \geq \frac{\delta(\beta m + 1)}{m} \Leftrightarrow x_2 \geq \frac{\delta - \beta m(1-\delta)}{m}.$$

To maximize her utility, a skilled proposer sets $x_2 = \max\left\{0, \frac{\delta - \beta(n-1)(1-\delta)}{n-1}\right\}$ when $m > 1$ and keeps the whole surplus when $m = 1$. This proposal is accepted by all players. The expected utilities are

$$\overline{u}_h = \frac{\beta m + 1}{m} \text{ and } \overline{u}_l = \beta.$$

Case 2: $1/(n-m) < \beta \leq 1$. Notice that this case is not possible when $m = n-1$, so we only consider $m < n-1$. By Lemma 4, any unskilled proposer keeps $1 - mx_3$ for herself and puts x_3 for each skilled player, where $x_3 \in [0, 1/m]$. On the other hand, by Lemma 5 any skilled proposer keeps $1-(m-1)x_4$ for herself and gives x_4 to each skilled responder (if $m > 1$), where $x_4 \in [0, 1/(m-1)]$. Following the same arguments as above, the accepted proposals that maximize proposers' utility are[9]

$$\begin{aligned} x_3 &= \left\{ \begin{array}{ll} \frac{\delta(1+\beta m)}{m(\delta - \beta\delta(n-m)+\beta n)} & \text{if } m \in \{2,...,n-2\} \\ \frac{\delta}{\delta + n(1-\delta)} & \text{if } m = 1 \end{array} \right. \quad \text{and} \\ x_4 &= \max\left\{0, \frac{\delta(1+\beta m)(\delta+\beta m)}{m(\delta - \beta\delta(n-m)+\beta n)} - \beta\right\} \text{ if } m \in \{2,...,n-2\}. \end{aligned}$$

Given these proposals, the equilibrium utilities are

$$\overline{u}_h = \left\{ \begin{array}{ll} \frac{(1+\beta m)(\delta+\beta m)}{m(\delta-\beta\delta(n-m)+\beta n)} & \text{if } m \in \{2,...,n-2\} \\ \frac{1+\beta}{n-\delta(n-1)} & \text{if } m = 1 \end{array} \right.$$

and

$$\overline{u}_l = \left\{ \begin{array}{ll} \frac{(1+\beta m)\beta}{\delta-\beta\delta(n-m)+\beta n} & \text{if } m \in \{2,...,n-2\} \\ \frac{1+\beta-\delta}{n-\delta(n-1)} & \text{if } m = 1 \end{array} \right. .$$

Case 3: $\delta/n < \beta \leq 1/(n-m)$. By Lemma 4, any unskilled proposer keeps $1 - mx_5 - (n-m-1)x_6$ for herself and puts x_5 for each skilled player and x_6 for each unskilled responder (if $m < n-1$), where $1 - mx_5 - (n-m-1)x_6, x_5, x_6 \geq 0$. On the other hand, Lemma 5 implies that any skilled proposer keeps $1-(m-1)x_7$ for

[9] Note that x_i's are not defined for all values of m, since there are some partitions where these proposals are never made. In the following, we specify this fact after their definition.

herself and gives x_7 to any other skilled player (if $m > 1$), where $x_7 \in [0, 1/(m-1)]$. The accepted proposals that maximize proposers' utility are

$$
\begin{aligned}
x_5 &= \frac{\delta}{n - \delta(n-m)}, \\
x_6 &= \frac{\delta(1-\delta)}{n - \delta(n-m)} \text{ if } m \in \{1, ..., n-2\}, \text{ and} \\
x_7 &= \max\left\{0, \frac{\delta - \beta n(1-\delta)}{n - \delta(n-m)}\right\} \text{ if } m \in \{2, ..., n-1\}.
\end{aligned}
$$

The expected equilibrium utilities are

$$
\overline{u}_h = \frac{\beta m + 1}{n - \delta(n-m)} \text{ and } \overline{u}_l = \frac{\beta m + 1 - \delta}{n - \delta(n-m)}.
$$

Case 4: $\beta \leq \delta/n$. By Lemma 4, any unskilled proposer keeps $1 - mx_8 - (n-m-1)x_9$ for herself and puts x_8 for each skilled player and x_9 for other unskilled (if $m < n-1$), where $1 - mx_8 - (n-m-1)x_9, x_8, x_9 \geq 0$. Lemma 5 implies that any skilled proposer keeps $1 - (m-1)x_{10} - (n-m)x_{11}$ for herself and gives x_{10} to other skilled agents (if $m > 1$) and x_{11} to unskilled agents, where $1 - (m-1)x_{10} - (n-m)x_{11}, x_{10}, x_{11} \geq 0$. The accepted proposals that maximize proposers' utility are

$$
\begin{aligned}
x_8 &= \frac{\delta}{n(1 - \beta(n-m))}, \\
x_9 &= \frac{\delta(1-\beta n)}{n(1 - \beta(n-m))} \text{ if } m \in \{1, ..., n-2\}, \\
x_{10} &= \begin{cases} \frac{\delta - \beta n(1-\delta)}{n(1-\beta)} & \text{if } m = n-1 \\ \frac{\delta(1+\beta n) - \beta n}{(1+\beta)n(1-\beta(n-m))} & \text{if } m \in \{2, ..., n-2\} \end{cases}, \text{ and} \\
x_{11} &= \frac{\delta - \beta n}{n(1 - \beta(n-m))}.
\end{aligned}
$$

Hence, the equilibrium expected utilities are

$$
\overline{u}_h = \frac{\beta m + 1}{(1 - \beta(n-m))n} \text{ and } \overline{u}_l = \frac{1}{n}.
$$

B.1.3. $m = 0$

In this case the proposer keeps $1 - (n-1)x_{12}$ for herself and puts x_{12} for each other player, where $x_{12} \in [0, 1/(n-1)]$. Non-proposers accept the proposal if and only if $x_{12} \geq \delta/n$. To maximize her utility, the proposer sets $x_{12} = \delta/n$. This proposal is accepted by all players. Therefore, $\overline{u}_l = 1/n$.

Next tables summarize the (symmetric) equilibrium expected utilities for any possible m.

$\overline{u}_h$	$\beta > 1$	$\beta \in (1/(n-m), 1]$	$\beta \in (\delta/n, 1/(n-m)]$	$\beta \leq \delta/n$
$m = n$	$\frac{\beta m+1}{m}$	$\frac{\beta m+1}{n}$	$\frac{\beta m+1}{n}$	$\frac{\beta m+1}{n}$
$m = n-1$	$\frac{\beta m+1}{m}$	$\frac{\beta m+1}{n-\delta}$	$\frac{\beta m+1}{n-\delta}$	$\frac{\beta m+1}{(1-\beta)n}$
$m \in \{2, ..., n-2\}$	$\frac{\beta m+1}{m}$	$\frac{(1+\beta m)(\delta+\beta m)}{m(\delta-\beta\delta(n-m)+\beta n)}$	$\frac{\beta m+1}{n-\delta(n-m)}$	$\frac{\beta m+1}{(1-\beta(n-m))n}$
$m = 1$	$\frac{\beta m+1}{m}$	$\frac{\beta+1}{n-\delta(n-1)}$	$\frac{\beta+1}{n-\delta(n-1)}$	$\frac{\beta+1}{(1-\beta(n-1))n}$
$m = 0$	—	—	—	—

$\overline{u}_l$	$\beta > 1$	$\beta \in (1/(n-m), 1]$	$\beta \in (\delta/n, 1/(n-m)]$	$\beta \leq \delta/n$
$m = n$	—	—	—	—
$m = n-1$	β	$\frac{\beta m+1-\delta}{n-\delta}$	$\frac{\beta m+1-\delta}{n-\delta}$	$\frac{1}{n}$
$m \in \{2, ..., n-2\}$	β	$\frac{(1+\beta m)\beta}{\delta-\beta\delta(n-m)+\beta n}$	$\frac{\beta m+1-\delta}{n-\delta(n-m)}$	$\frac{1}{n}$
$m = 1$	β	$\frac{1+\beta-\delta}{n-\delta(n-1)}$	$\frac{1+\beta-\delta}{n-\delta(n-1)}$	$\frac{1}{n}$
$m = 0$	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$

B.2. Asymmetric case

By Lemma 4 we know that when $m \in \{1, ..., n-2\}$ and $\beta > 1/(n-m)$, any unskilled proposer prefers to indirectly compensate other unskilled by giving a sufficient share of the budget to skilled players. In the symmetric case, this compensation is evenly split among all skilled agents. Now, we consider the opposite extreme case where all unskilled proposers compensate unskilled responders through a unique skilled player called the *preferential* player, p.

Notice that the asymmetric compensations described above require two skilled and two unskilled agents, at least. So, asymmetries among skilled agents will not arise when $m \in \{0, 1, n-1, n\}$. Therefore, we only need to consider the cases where $m \in \{2, ..., n-2\}$ and $\beta > 1/(n-m)$. Two subcases need to be distinguished at this point.

B.2.1. $\beta > 1$

In this case, any unskilled proposer gives the whole budget to the preferential skilled player and any skilled proposer gives no share of the budget to unskilled responders. So, the preferential player keeps $1-(m-1)x_{13}$ for herself and puts $x_{13} \in [0, 1/(m-1)]$ for any other skilled player. On the other hand, any ordinary skilled proposer keeps $1-x_{14}-(m-2)x_{15}$ for herself and puts x_{14} for the preferential player and x_{15} for the rest of skilled agents, where $1-x_{14}-(m-2)x_{15}, x_{14}, x_{15} \geq 0$. When the proposer is the preferential player, skilled responders weigh their utility from the current proposal, $x_{13}+\beta$, against the expected utility from having a new proposer selected following a delay, $\frac{\delta(1+(n-m+1)\beta-(m-2)x_{15}-x_{14})}{n-\delta(m-1)}$. Thus, they accept the proposal if and only if

$$x_{13}+\beta \geq \frac{\delta(1+(n-m+1)\beta-(m-2)x_{15}-x_{14})}{n-\delta(m-1)}.$$

On the other hand, when the proposer is an ordinary skilled player the following two conditions must hold. First, the preferential player weighs her utility from the current proposal, $x_{14}+\beta$, against her expected utility from having a new proposer selected following a delay, $\frac{\delta(n-m+1)(1+\beta)-\delta(m-1)x_{13})}{n-\delta(m-1)}$. Thus, she accepts the proposal if and only if

$$x_{14}+\beta \geq \frac{\delta(n-m+1)(1+\beta)-\delta(m-1)x_{13})}{n-\delta(m-1)}.$$

Second, the other skilled responders weigh their utility from the current proposal, $x_{15}+\beta$, against the expected utility from having a new proposer selected following a delay, $\frac{\delta(1+(n-m+1)\beta-(m-2)x_{15}-x_{14})}{n-\delta(m-1)}$. Thus, these responders accept the proposal if and only if

$$x_{15}+\beta \geq \frac{\delta(1+(n-m+1)\beta-(m-2)x_{15}-x_{14})}{n-\delta(m-1)}.$$

The above three conditions imply

$$\begin{aligned}
x_{13} &\geq (1-\delta)\left(\frac{\delta}{n-\delta m}-\beta\right), \\
x_{14} &\geq \frac{\delta(n-m+1-\delta)}{n-\delta m}-\beta(1-\delta), \text{ and} \\
x_{15} &\geq (1-\delta)\left(\frac{\delta}{n-\delta m}-\beta\right).
\end{aligned}$$

Given that $x_{13}, x_{14}, x_{15} \geq 0$, in order to maximize their utility proposers set $x_{13}=x_{15}=0$ and

$$x_{14}=\max\left\{0, \frac{\delta(n-m+1-\delta)}{n-\delta m}-\beta(1-\delta)\right\}.$$

These proposals are accepted by all players. Their expected utilities are

$$\overline{u}_l=\beta, \overline{u}_p=\frac{(1+\beta)(n-m+1)}{n-\delta m+\delta} \text{ and } \overline{u}_h=\beta+\frac{(1+\beta)(1-\delta)}{n-\delta m+\delta}$$

when $x_{14}>0$; and

$$\overline{u}_l=\beta, \overline{u}_p=\beta+\frac{n-m+1}{n} \text{ and } \overline{u}_h=\beta+\frac{1}{n},$$

otherwise.

B.2.2. $\beta \in (1/(n-m), 1]$

Remember that by Lemma 4, any unskilled proposer prefers to give no share of the surplus to other unskilled when $\beta > 1/(n-m)$. So, she keeps $1-x_{16}-(m-1)x_{17}$ for herself and puts x_{16} for the preferential player and x_{17} for the rest of skilled players, where $1-x_{16}-(m-1)x_{17}, x_{16}, x_{17} \geq 0$. On the other hand, we claim that in this asymmetric case, skilled proposers do not need to give any share of the budget to unskilled responders. So, (i) the preferential player keeps $1-(m-1)x_{18}$ for herself and puts x_{18} for the rest of skilled agents and (ii) any ordinary skilled player keeps $1-x_{19}-(m-2)x_{20}$ for herself and puts x_{19} for the preferential player and x_{20} for other skilled responders, where $x_{18} \in [0, 1/(m-1)]$ and $1-x_{19}-(m-2)x_{20}, x_{19}, x_{20} \geq 0$. Following the same steps as above, the accepted proposals that maximize proposers' utilities imply:

$$\begin{aligned}
x_{16} &= \frac{\delta(1+\beta m)(n-\delta(m-1))}{n(\delta(1+\beta m)+\beta n(1-\delta))}, \\
x_{17} &= \frac{\delta^2(1+\beta m)}{n(\delta-\beta\delta(n-m)+\beta n)}, \\
x_{18} &= x_{20} = \max\left\{0, \frac{\delta^2(1+\beta m)-\beta^2 n^2(1-\delta)}{n(\delta-\beta\delta(n-m)+\beta n)}\right\}, \text{ and} \\
x_{19} &= \max\left\{0, \frac{\delta^2(1+\beta m)(n-m+1)-\beta^2 n^2(1-\delta)}{n(\delta-\beta\delta(n-m)+\beta n)}\right\}.
\end{aligned}$$

The expected equilibrium utilities are $\overline{u}_l = \frac{(1+\beta m)\beta}{\delta-\beta\delta(n-m)+\beta n}$ and

$$\overline{u}_p = \frac{(1+\beta m)(\beta n+\delta(n-m+1))}{n(\delta-\beta\delta(n-m)+\beta n)} \text{ and } \overline{u}_h = \frac{(1+\beta m)(\beta n+\delta)}{n(\delta-\beta\delta(n-m)+\beta n)}$$

when $x_{18} = x_{20} > 0$ and $x_{19} > 0$;

$$\overline{u}_p = \frac{\delta(1+\beta m)(1+\beta-(1-\delta)m)+(1-\delta)(\beta+\beta^2+\delta+\beta\delta m)n}{(n-(n-1)\delta)(\delta-\beta\delta(n-m)+\beta n)} \text{ and}$$

$$\overline{u}_h = \frac{(1+\beta-\delta)\delta(1+\beta m)+\beta(1-\delta)(1+\beta+\beta m)n}{(n-(n-1)\delta)(\delta-\beta\delta(n-m)+\beta n)}$$

when $x_{18} = x_{20} = 0$ and $x_{19} > 0$; or

$$\overline{u}_p = \frac{(1+\beta m)(\delta+\delta(\beta+\delta-1)m+(1-\delta)(\beta+\delta)n)}{(n-\delta(n-m))(\delta-\beta\delta(n-m)+\beta n)} \text{ and}$$

$$\overline{u}_h = \frac{1+\beta m}{n(1-\delta)+\delta m}$$

when $x_{18} = x_{20} = x_{19} = 0$. To prove the claim we must check that $\delta\overline{u}_l < \beta m$. This holds for any pair (β, δ) such that $\beta \in (1/(n-m), 1]$ and $\delta \in (0, 1)$.

Notice that there exists a $\overline{\delta} < 1$ such that if $\delta > \overline{\delta}$ then $x_{14}, x_{18}, x_{19}, x_{20} > 0$. The following table summarizes the new equilibrium utilities for this extreme asymmetric case for $\delta > \overline{\delta}$:

$m \in \{2, ..., n-2\}$	$\beta > 1$	$\beta \in (1/(n-m), 1]$
$\overline{u}_p$	$\frac{(1+\beta)(n-m+1)}{n-\delta m+\delta}$	$\frac{(1+\beta m)(\beta n+\delta(n-m+1))}{n(\delta-\beta\delta(n-m)+\beta n)}$
$\overline{u}_h$	$\beta + \frac{(1+\beta)(1-\delta)}{n-\delta m+\delta}$	$\frac{(1+\beta m)(\beta n+\delta)}{n(\delta-\beta\delta(n-m)+\beta n)}$
$\overline{u}_l$	β	$\frac{(1+\beta m)\beta}{\delta-\beta\delta(n-m)+\beta n}$

C. Proof of Proposition 6

Asymmetries among skilled agents can only arise when unskilled proposers overcompensate skilled responders in order to gain the support of the other unskilled agents. By Lemma 4, this occurs when $\beta > 1/(n-m)$. From this it trivially follows that when $\beta < 1/(n-2)$ or $n = 2$ the SSPE coincide with what is stated in Propositions 4 and 5. So, next we focus on $\beta \geq 1/(n-2)$ and $n > 2$.

Proof. Trivially, when $m \in \{0, 1\}$ there cannot be asymmetries among skilled agents. Moreover, when $m \in \{n-1, n\}$, unskilled proposers (if any) do not need to gain the support of another unskilled, so asymmetries cannot arise, either. Consequently, the equilibrium conditions when $m \in \{0, n\}$ do not change with respect to the symmetric case analyzed in Proposition 5. Furthermore,

$$\beta > 1/(n-m) \Leftrightarrow m < (\beta n - 1)/\beta \equiv \widehat{m},$$

so asymmetries cannot arise for any $m \geq \widehat{m}$ (notice that $\beta \geq 1/(n-2)$ implies $\widehat{m} \geq 2$). By Proposition 5, m skilled agents can be sustained in a symmetric equilibrium if and only if $c \in \left[\frac{1}{m+1}, \frac{1}{m}\right]$. Consequently, for a given β, any $m \geq \widehat{m}$ can be sustained in equilibrium under the same conditions stated by Proposition 5 whenever $c \leq \beta/(\beta n - 1)$, even though there is a preferential partner.

Following the previous reasonings, we can conclude that the SSPE with a preferential partner yields $m^* = n$ if $c \leq 1/n$, $m^* \in \{2, ..., n-1\}$ if $c \in (1/n, \beta/(\beta n - 1)]$ where m^* satisfies $c \in \left[\frac{1}{m^*+1}, \frac{1}{m^*}\right]$, and $m^* = 0$ if $c > (n(1+\beta) - 1)/n$. Now, it remains to know under which conditions (if any) $m^* = 1$ can be sustained in an SSPE with a preferential partner and analyze the cases where $m < \widehat{m}$. In the following, we use repeatedly the bargaining expected utilities provided in Proposition 2.

First, consider that $m = 1$. This partition can be sustained in equilibrium if (i) the skilled agent does not want to become unskilled; *i.e.,* (as in the symmetric case)

$$c \leq \frac{n(1+\beta) - 1}{n} \text{ for any } \beta > \frac{1}{n};$$

and (ii) no unskilled agent prefers to be an ordinary skilled agent,[10] so

$$c \geq 0 \text{ when } \beta > 1 \text{ or } c \geq \frac{1}{n} \text{ when } \beta \in \left(\frac{1}{n-2}, 1\right].$$

Therefore, $m^* = 1$ if and only if

$$c \leq \frac{n(1+\beta)-1}{n} \text{ when } \beta > 1,$$

and

$$c \in \left[\frac{1}{n}, \frac{n(1+\beta)-1}{n}\right] \text{ when } \beta \in \left(\frac{1}{n-2}, 1\right].$$

Notice that $\beta/(\beta n - 1) < (n(1+\beta)-1)/n$ for any $\beta \geq 1/(n-2)$.

Second, any $m \in \{2, ..., n-2\}$ can be sustained in equilibrium when (i) no ordinary skilled agent has incentives to become unskilled;[11] *i.e.,*

$$\beta - \beta + c \leq 0 \Rightarrow c \leq 0 \text{ when } \beta > 1,$$

$$\beta - \frac{\beta n + 1}{n} + c \leq 0 \Rightarrow c \leq \frac{1}{n} \text{ when } \beta \in \left(\frac{1}{n-m}, 1\right],$$

$$\beta - \frac{\beta m + 1}{m} + c \leq 0 \Rightarrow c \leq \frac{1}{m} \text{ when } \beta \in \left[\frac{1}{n-2}, \frac{1}{n-m}\right];$$

and (ii) no unskilled agent prefers to be an ordinary skilled agent. So, if $m \in \{2, ..., n-3\}$ then

$$\beta - \beta - c \leq 0 \Rightarrow c \geq 0 \text{ when } \beta > 1,$$

$$\frac{\beta n + 1}{n} - \beta - c \leq 0 \Rightarrow c \geq \frac{1}{n} \text{ when } \beta \in \left(\frac{1}{n-m-1}, 1\right],$$

$$\frac{\beta(m+1)+1}{m+1} - \beta - c \leq 0 \Rightarrow c \geq \frac{1}{m+1} \text{ when } \beta \in \left[\frac{1}{n-2}, \frac{1}{n-m-1}\right],$$

and, if $m = n-2$ then

$$\beta\frac{(n-1)+1}{n-1} - \beta - c \leq 0 \Rightarrow c \geq \frac{1}{n-1} \text{ when } \beta \geq \frac{1}{n-2}.$$

Hence, $m^* = \{2, ..., n-3\}$ if and only if

$$c = \frac{1}{n} \text{ when } \beta \in \left(\frac{1}{n-m^*-1}, 1\right]$$

[10] We assume that an unskilled agent cannot become the preferential agent except when $m = 0$.

[11] Notice that if ordinary skilled agents do not prefer to become unskilled neither the preferential agent prefers it.

or, as in the symmetric case, $m^* \in \{2, ..., n-2\}$ if and only if

$$c \in \left[\frac{1}{m^*+1}, \frac{1}{m^*}\right] \text{ when } \beta \in \left[\frac{1}{n-2}, \frac{1}{n-m^*}\right].$$

Notice that $c \leq 1/m$ and $\beta \leq 1/(n-m)$ implies $c \leq \beta/(\beta n - 1)$.

Finally, consider $m = n-1$. Notice that $n - 1 < \widehat{m}$ if and only if $\beta > 1$. So, if $\beta \leq 1$ then $m^* = n-1$ will be sustained under the same conditions of the symmetric case. When $\beta > 1$, $m = n-1$ would be sustained in equilibrium if (i) no skilled agent prefers to be unskilled; *i.e.*,

$$\beta - \frac{\beta(n-1)+1}{n-1} + c \leq 0 \Rightarrow c \leq \frac{1}{n-1};$$

and (ii) the unskilled agent does not prefer to acquire skills; *i.e.*,

$$\frac{\beta n + 1}{n} - \beta - c \leq 0 \Rightarrow c \geq \frac{1}{n}.$$

Thus, when $\beta > 1$, $m^* = n-1$ if and only if $c \in \left[\frac{1}{n}, \frac{1}{n-1}\right]$. Notice that when $\beta > 1$, $1/n < \beta/(\beta n - 1) < 1/(n-1)$. ■