Partial collusion in an asymmetric duopoly

Marc Escrihuela-Villar[*]

Universitat de les Illes Balears

February, 2012

Abstract

In this paper we investigate the connection between cost asymmetries and the sustainability of collusion within the context of a infinitely repeated Cournot duopoly. We assume that firms are able to coordinate on distinct output levels than the unrestricted joint profit maximization outcome. We show that, in our model, regardless of the degree of cost asymmetry, at least some collusion is always sustainable if firms are patient enough. We also endogenize the degree of collusion and show that it has an upper bound determined by the most inefficient firm.

JEL Classification: L11; L13; L41; D43

Keywords: Collusion; Sustainability; Asymmetry

[*]Corresponding author. Mailing address: Departamento de Economia Aplicada. Edificio Jovellanos Ctra. Valldemossa km 7.5 07122 Palma de Mallorca Baleares - España. Tel. +34 971173242. Fax: +34 971172389. Email: marc.escrihuela@uib.es

1 Introduction

The analysis of cartel formation and collusion sustainability in oligopoly markets has a long tradition in the economic literature. Following Friedman's (1971) approach, and the subsequent contributors to the subject, firms can maintain collusive agreements by means of the threat of reversion to comprehensive noncooperation in the event that deviation occurs. The punishment strategy which underpins this result is that firms weigh the short-term gains from deviation against the losses which subsequently arise from punishment. If the latter exceed the former, then deviation is deterred. In this literature, initially the assumption was that all firms were identical in terms of their costs, even if there was a degree of differentiation amongst their products or in firm's timing decision.[1] The intuition behind this assumption can be inferred from an early paper by Patinkin (1947), where a cartel maximizes total industry profits and therefore allocates output quotas so that the marginal cost is the same for all firms. Then, the cartelized industry operates as if it was a multiplant monopolist allocating output between plants. However, costs asymmetries are proved to play an important role when firms attempt to reach collusion. As Bain (1948) points out, costs heterogeneity would mean that, in the absence of side payments between firms, such an allocation might not be viable as inefficient firms may obtain lower profits in the cartel than in the non-cooperative equilibrium. The intuition is that firms may find it difficult to agree to a common collusive policy because firms with a lower marginal cost will insist in lower prices than what the other firms would wish to sustain. More generally, the common wisdom is that the diversity of cost structures may rule out any possible agreement in pricing policies and so exacerbate coordination problems. In addition, technical efficiency would require allocating production quotas to low-cost firms, but this would clearly be difficult to sustain in the absence of explicit agreements and side transfers.

More recently, the possibility of firms operating with different cost functions has

---

[1]An exception to this approach can be found in the literature of partial cartels with a dominant firm (namely, a cartel) facing a competitive fringe. In this type of model, as Donsimoni (1986) has shown, a degree of cost heterogeneity can be accommodated.

received more attention. For instance, Osborne and Pitchik (1983), in a static non-cooperative model were firms are capacity-constrained allow for side payments and show that the profit per unit of capacity of the small firm is higher than that of the large one. Schmalense (1987) in a static game with linear cost in a Cournot setting characterizes the set of profit vectors by applying a number of selection criteria such as the Nash bargaining solution. He finds that if a leading firm's cost advantage is substantial, its potential gains from collusion are relatively small. By their very nature, however, in a static model cartel members do not cheat on a cartel agreement since it is assumed that agreements are sustained through binding contracts. This may therefore, be viewed as a model of explicit or binding collusion. These papers thus do not impose the incentive compatibility constraints of subgame perfection and the collusive outcome derived in their models may not be self-enforced. On the other hand, following the supergame-theoretic approach to collusion of Friedman, Rotschild (1999) shows that the stability of the cartel may depend crucially upon the relative efficiencies of the firms and remarks that joint profit maximization becomes less likely as cost functions differ between firms. Vasconcellos (2005), in a quantity setting oligopoly model, assumes asymmetry by assuming that firms have different shares of a specific asset and shows that the sustainability of perfect collusion crucially depends on the most inefficient firm in the agreement, which represents the main obstacle to the enforcement of collusion. Summarizing, both the literature on static cartel stability and the dynamic models of tacit collusion suggest that collusion is unlikely to be observed in the presence of substantial competitive advantage, and therefore, a prior step before studying collusion sustainability when costs are heterogeneous and firms agree on output quotas is to consider whether collusion is viable. On the other hand, the analysis of the empirical literature also indicates that cost asymmetries hinder collusion (see for instance Levenstein and Suslow (2006)).[2]

To our knowledge a neglected question is whether partial collusion can be a way to sustain a collusive agreement in the presence of cost asymmetry. Partial collusion is often referred to as coordination on distinct output or price levels than the joint profit max-

---

[2]Mason, Phillips and Nowell (1992) show that in an experimental duopoly game cooperation is also more likely when players face symmetric production costs.

imization outcome.[3] The main aim of the present paper is thus twofold (i) we study if cost heterogeneity is sufficient to make it impossible for producers to collude when coordination is not necessarily on the allocation that maximizes total industry profits. In this sense, in their empirical studies Eckbo (1976) and Griffin (1989) provide an interesting motivation with this respect by finding that cartels that are made up of similar sized firms are more able to raise price. Concretely Eckbo obtains that in some cases high cost members of a cartel may produce at a cost larger than 50% above low-cost members. Consequently, a natural question arises about why and to what extent should cost asymmetries be a restraint for collusion. And (ii) we also study how the degree of collusion can be endogenously determined.

We develop a multi-period duopoly model with two cost-asymmetric firms producing a quantity of a homogeneous product. We use subgame perfect Nash equilibrium — henceforth, SPNE— as solution concept. It is well known that our repeated game setting exhibits multiple SPNE collusive agreements. Therefore, to select among those equilibria we adopt the particular criterion of restricting strategies to grim "trigger strategies". We assume also that firms maximize the summation of its own profits and a proportion of the profits of the other firm. As a consequence, this proportion is considered as the degree of collusion which implies that firms can coordinate their output even when the joint profit maximization agreement does not correspond to a SPNE of the repeated game.[4] The notion of collusion employed thus resembles the one in the models of Reynolds and Snapp (1986) and Farrell and Shapiro (1990a) where the present formulation gets part of its inspiration. These papers have considered the competitive effects of partial ownership of rival firms. In the context of a single-period Cournot oligopoly model, they show that, as the degree of cross ownership among rivals increases, the equilibrium in the market becomes less competitive in the sense that aggregate output falls toward the monopoly level.

We obtain that, in our model, if firms are patient enough at least some collusion can

---

[3]See for instance Verboven (1997) or Escrihuela-Villar (2008).

[4]As shown by the famous Folk Theorem, any combination of individually rational profits is sustainable as a SPNE if firms are sufficiently patient (Fundenberg and Maskin 1986).

always be sustained regardless of the cost heterogeneity. The intuition is that even tough cost asymmetry hinders collusion firms can always coordinate on an output level below the competitive one provided that their discount factor is high enough. The main implication of this result is that we can expect collusion between firms to occur, at least to some extent, also in very asymmetric markets as long as firms are able to coordinate on distinct levels than the unrestricted joint profit maximization outcome. We obtain also that the endogenous degree of collusion to be sustained has an upper bound that can never be overcome which is determined by the most inefficient firm. Finally, a welfare analysis shows that, surprisingly enough, consumer surplus can increase with cost inefficiency as long as increasing cost heterogeneity implies a lower degree of collusion.

The remainder of the paper is organized as follows. In Section 2, we present the model. In Section 3, we analyze the endogenous degree of collusion. In two subsections a welfare analysis and a numerical simulation to illustrate the results are presented. We conclude in Section 4. All proofs are grouped together in the appendix.

2   The model

We consider an industry with two asymmetric firms indexed by $i = 1, 2$. Each firm produces a quantity of a homogeneous product with a quadratic cost function $c_i(q_i) = c_i q_i^2$ where q_i is the output produced by firm i. We assume that firms simultaneously choose quantities and without loss of generality we assume $c_1 > c_2$. The industry inverse demand is given by the piecewise linear function

$$p(Q) = \max(0, 1 - Q),$$

where $Q = q_1 + q_2$ is the industry output and p is the output price. We assume that firms compete repeatedly over an infinite horizon with complete information (i.e. each of the firms observes the whole history of actions) and discount the future using a discount factor $\delta \in (0, 1)$. Time is discrete and dates are denoted by $t = 1, 2, ...$ In this framework, a pure strategy for firm i is an infinite sequence of functions $\{S_i^t\}_{t=1}^{\infty}$ with $S_i^t : \sum^{t-1} \longrightarrow \mathcal{Q}$ where $\sum^{t-1}$ is the set of all possible histories of actions (output choices) of both firms up

to $t-1$, with typical element σ_j^τ, $j = 1, 2$, $\tau = 1, ..., t-1$, and $\mathcal{Q}$ is the set of output choices available to each firm. Following Friedman (1971), we restrict our attention to the case where each firm is only allowed to follow grim trigger strategies. In words, these strategies are such that firms adhere to a collusive agreement until there is a defection, in which case they revert forever to the static Cournot equilibrium. Let q_i^n and q_i^c denote the output corresponding to Cournot noncooperative and the collusive output respectively. Since we restrict attention to trigger strategies, $\{S_i^t\}_{t=1}^\infty$ can be specified as follows. At $t = 1$, $S_i^1 = q_i^c$, while at $t = 2, 3, ...$

$$S_i^t(\sigma_j^\tau) = \begin{cases} q_i^c \text{ if } \sigma_j^\tau = q_j^c \text{ for all } j = 1, 2, \ \tau = 1, ..., t-1 \\ q_i^n \text{ otherwise.} \end{cases} \tag{1}$$

The profit function for firm i is given by

$$\Pi_i(q_i, q_j) = (a - Q)q_i - c_i(q_i)^2$$

As shown by Friedman (1971), firms producing q_1^c and q_2^c in each period can be sustained as a SPNE of the repeated game with the strategy profile (1) if and only if for given values of c_1 and c_2 and δ, the following conditions are satisfied

$$\frac{\Pi_i(q_i^c, q_j^c)}{1 - \delta} \geq \Pi_i^d(q_j^c) + \frac{\delta \Pi_i^n}{1 - \delta} \text{ for } i, j = 1, 2 \text{ and } j \neq i. \tag{2}$$

where $\Pi_i^d(q_j^c)$ denotes the profits attained of firm i by an optimal deviation from a collusive output of firm j, q_j^c, and Π_i^n denotes the Cournot equilibrium profits for firm i. Multiplicity of equilibria is obtained since condition (2) is satisfied for different collusive outputs. To select among such equilibria, we consider a particular model of partial collusion where firms maximize its own profits and also take into account to some extent the profits of the other firm. In other words, firm 1 maximizes $\Pi_1(q_1, q_2) + \alpha \Pi_2(q_1, q_2)$ and firm 2 maximizes $\Pi_2(q_1, q_2) + \alpha \Pi_1(q_1, q_2)$ where $\alpha \in [0, 1]$. Consequently, α may represent the degree of collusion, that for simplicity, we assume to be symmetric. Therefore, the present model encompasses the Cournot case if $\alpha = 0$ and the profit maximizing allocation with full collusion if $\alpha = 1$. It is a standard exercise to obtain that the equilibrium quantities for a given α are $q_1^c = \frac{1 + 2c_2 - \alpha}{4c_2 + 4c_1(1 + c_2) - (-1 + \alpha)(3 + \alpha)}$ and $q_2^c = \frac{1 + 2c_1 - \alpha}{4c_2 + 4c_1(1 + c_2) - (-1 + \alpha)(3 + \alpha)}$. We note that q_1^c

and q_2^c decrease with α. Intuitively, as α increases firms are more able to coordinate their behavior, and consequently mimic cartel behavior cutting their production to increase price. Then, firms producing respectively q_1^c and q_2^c is a SPNE of the repeated game only if δ exceeds a certain critical level, such that both conditions described in (2) are satisfied. We denote by $\bar{\delta}_i$ the critical value above which the condition for firm i is satisfied. Thus, the condition for firms producing q_1^c and q_2^c to be a SPNE of the repeated game becomes $\delta \geq \max\{\bar{\delta}_1, \bar{\delta}_2\}$. It is well known that when firms maximize joint profits (see for instance Rotschild (1999)) the condition on δ in 2 is more easily satisfied for the most efficient firm $(\bar{\delta}_1 > \bar{\delta}_2$ if $\alpha = 1)$. In our case, as we prove in the appendix that this is also true for all possible values of α. Consequently, in order to characterize the equilibrium is enough to consider $\bar{\delta}_1$:

Definition 1 *Collusion is said to be partial if $\alpha \in (0,1)$. Then, partial collusion is sustainable if $\delta \geq \bar{\delta}_1$.*

To simplify the expressions and since we are interested in analyzing the effect of firms' cost asymmetry in collusion performance, we assume throughout the paper that $c_2 = 1$ and $c_1 > 1$ and therefore firm 1 can be considered as the inefficient firm and firm 2 as the efficient firm. It can be verified that $\bar{\delta}_1$ is a function of α and c_1:

$$\bar{\delta}_1(\alpha, c_1) = \frac{(7 + 8c_1)^2 \alpha(-1 - 2c_1 + \alpha)^2}{(5 - 2c_1(-2 + \alpha) - \alpha)(84 + 96c_1^2 - \alpha(19 + 13\alpha) - 2c_1(-90 + \alpha(10 + 7\alpha)))}.$$

It is worthwhile noting that our model exhibits equilibria where coordination on distinct output levels exists depending on the extent to which firms coordinate their actions (namely, α). We can also check that $\bar{\delta}_1(\alpha, c_1)$ increases both with c_1 and α, which is very intuitive, since collusion becomes harder to sustain as cost asymmetry increases and as the degree of collusion to be sustained increases. We can now state the following Proposition.

Proposition 1 *When $c_1 > 1$, there always exists $\alpha \in (0,1)$ such that $\bar{\delta}_1(\alpha, c_1) < 1$.*

Proposition 1 establishes that if firms are patient enough partial collusion can always be sustained regardless of the cost asymmetry of firms. The intuition behind this result is fairly simple. It is well known in the literature on collusion that when firms' cost

asymmetry increases collusion becomes harder to sustain.[5] However, in this case, if firms conform to sustain "less" collusion, there is always a small enough degree of collusion such that when firms are patient enough this collusion can be sustained. For instance, in the standard case of firms maximizing joint profits ($\alpha = 1$), if $c_1 > 1.549$ then $\bar{\delta}_1(\alpha, c_1) > 1$. Consequently, the standard joint profit maximization allocation cannot be sustained. However, when we consider partial collusion the picture changes. For example, if $c_1 = 2$, partial collusion corresponding to $\alpha \leq 0.5$ can be sustained if $\delta \geq 0.65$ (since $\bar{\delta}_1(0.5, 2) = 0.65$). An implication of this result is that collusion between very asymmetric firms can also be sustained when partial collusion or coordination on other output levels different from the joint profit-maximizing allocation is considered.

3   Endogenous partial collusion

So far we have only considered the critical level of the discount factor for a given value of α. It is also natural to consider the case where firms have the possibility to choose the degree of collusion or in other words, when α is endogenous. We add in this section an initial stage to the game we have considered so far, in which firms simultaneously choose the degree of collusion α. Then, the problem that firms face in this first stage of the game is the following where we denote by $\Pi_i^0(q_1, q_2)$ the profits attained by firm i with $j \neq i$ in the initial stage

$$\max_{\alpha} \Pi_i^0(q_1, q_2) = \begin{cases} \Pi_i(q_1, q_2) + \alpha \Pi_j(q_1, q_2) \text{ if } \delta \geq \bar{\delta}_1(\alpha, c_1) \\ \Pi_i(q_1, q_2) \qquad \text{otherwise} \end{cases} \tag{3}$$

Obviously if $\delta < \bar{\delta}_1(\alpha, c_1)$ no collusion is sustainable and firms' profits do not depend on α. Consequently, both firms would obtain the standard Cournot equilibrium profits. On the contrary, if $\delta \geq \bar{\delta}_1(\alpha, c_1)$ partial collusion is sustainable. By backward induction we obtain that after the initial stage of the game $q_1 = q_1^c$ and $q_2 = q_2^c$ and therefore firms

---

[5]For instance Harrington (1991) uses the Nash bargaining solution concept to obtain that, in general, the larger the cost differences, the higher the discount factor needed to sustain tacit collusion. More recently Ciarreta and Gutierréz-Hita (2011) reach similar conclusions when firms compete in supply functions.

obtain $\Pi_i^0(q_1^c, q_2^c)$ that depends on α. However, solving (3) non-cooperatively for both firms simultaneously, as it is formulated, is impossible since we assumed that the variable α is common to both firms but the following lemma will enable us to endogenize the common value of α.

Lemma 1 $\Pi_2(q_1^c, q_2^c)$ *is always increasing in α. Conversely, $\Pi_1(q_1^c, q_2^c)$ has a maximum value in the interval $\alpha \in [0, 1]$ at α^* with $0 < \alpha^* < 1$.*

Therefore, the most efficient firm is always better off when collusion is partial than without collusion and the best scenario for this firm is $\alpha = 1$. This is not the case, however, for the inefficient firm. Intuitively, in this case if α increases, it will pay for both firms to switch production from the inefficient to the efficient firm which is true for the inefficient firm just to certain extent. On the contrary, if $\alpha > \alpha^*$, the allocation rule of output implies that firms 1 produces "too little" and its profits are smaller than when $\alpha = \alpha^*$. It follows directly from Lemma 1 that whatever degree of coordination is going to be implemented in equilibrium when α is endogenous, it is enough to solve (3) for firm 1. The reason is that since $\bar{\delta}_1(\alpha, c_1) > \bar{\delta}_2$, the value of α solving (3) for firm 1 is the only potential mutual agreement on α in which both firms could adhere. Summarizing, the timing of the game we solve is thus the following. In the first stage of the game, for a given value of the discount factor and c_1, firm 1 maximizes $\Pi_1^0(q_1, q_2)$ with respect to α. Posteriorly, both firms repeatedly compete over an infinite horizon with a degree of collusion given by the α obtained in the first stage of the game. We can now state the following Proposition.

Proposition 2 *Let's consider the function $\bar{\delta}_1(\alpha, c_1) = \delta$, and $\bar{c}_1$ as a particular value of c_1 where $\delta^* \equiv \bar{\delta}_1(\alpha^*, \bar{c}_1)$. If $\delta < \delta^*$, the endogenous degree of collusion α is given by $\alpha = \bar{\delta}_1^{-1}(\delta, \bar{c}_1)$. If $\delta \geq \delta^*$, then $\alpha = \alpha^*$.*

Proposition 2 establishes that for a given value of the discount factor, (2) is a binding constraint for firm 1 and the endogenous degree of collusion is the α that solves it. This is true only to the extent that the degree of collusion that can be sustained is not beyond a certain point. The intuition behind is that if the discount factor is relatively small,

firms want to sustain the maximum possible degree of collusion that the value of the discount factor permits. This allows both firms to benefit from collusion. However, if the discount factor is larger than a certain threshold this is not true anymore. In this case, the inefficient firm does not want to sustain a large degree of collusion where an efficient allocation of output would force this firm to produce a relatively small quantity obtaining smaller profits than if $\alpha = \alpha^*$.

It is now natural to analyze how the endogenous degree of collusion varies with the discount factor and the cost parameter of the inefficient firm. This result is very intuitive.

Proposition 3 *The endogenous degree of collusion α decreases with c_1 and increases with δ if $\delta < \delta^*$. If $\delta \geq \delta^*$, it decreases with c_1 and does not depend on δ.*

We observe that firms' asymmetry hinders collusion sustainability and that when firms are more patient they can sustain a larger degree of collusion. This is true only up to a certain critical level since beyond this point, more collusion is not desirable anymore for the inefficient firm. Consequently, in this case, an increase in the discount factor does not help sustaining collusion.

3.1 Welfare

To study the welfare effects of partial collusion we use in this subsection the total consumer surplus as a welfare measure.[6] Thus, as it was to be expected, it is immediate to check that welfare decreases with the degree of collusion since firms sustaining a larger α is reflected in a contraction of output to increase price. However, it is more interesting to study the effect of firms asymmetry on welfare. For instance, at first glance it seems clear to expect that if c_1 increases welfare should decrease. But in the present case, we know from Proposition 3 that an increase in c_1 may also decrease the degree of collusion that can be sustained, thus benefiting consumers.

---

[6]Several recent papers call for antitrust agencies to use a consumer surplus standard rather than a total welfare standard (see for instance Pittman (2007)). Often, the argument against a consumer welfare standard is that it implies a tolerance for monopsony. However, it is well-known that this is more likely to occur in markets for intermediate goods and we focus here in a final good collusive market.

Proposition 4 *For a given value of the discount factor, welfare is maximized at a particular value of $c_1 \in (1, \infty)$.*

Intuitively, with a variation of c_1 we have two forces that act on opposite directions. An increase in c_1 implies that total output is produced in a less efficient way which reduces welfare. On the other hand, an increase in c_1 also hinders collusion sustainability enhancing welfare. Proposition 4 states thus that welfare is maximized in a particular degree of asymmetry. An interpretation of this result is that with partial collusion, technical efficiency may also harm welfare by allowing firms to sustain a larger degree of collusion. Another observation is that concentration, as measured by the Herfindahl index for instance, is a useful, though imperfect, guide to assessing the welfare effects of for instance a merger. In this sense it is commonly believed that concentration is perfectly negatively correlated with welfare.[7]

Corollary 1 *An increase in market concentration measured by the Herfindahl index may lead to an increase in welfare.*

An increase in firms' cost asymmetry obviously increases market concentration. At the same time, however, firms' capacity to collude is also reduced and consequently, as Proposition 4 shows, consumer surplus may increase.

3.2   Numerical Example

A numerical example is provided to illustrate the results of the present paper. We consider an industry where $c_1 = 1.01$. Consequently, the degree of collusion that maximizes the profits of the inefficient firm is $\alpha^* = 0.971$ (note that that if $c_1 = 1$, obviously $\alpha^* = 1$). Then, if for instance $\delta = 0.4$, since $\bar{\delta}_1(0.971, 1.01) = 0.506 < 0.4$, α^* cannot be sustained. As a consequence, we can obtain (Proposition 2) the endogenous degree of collusion for this value of the discount factor by solving the equation $\bar{\delta}_1(\alpha, 1.01) = 0.4$. The solution is $\alpha = 0.7359$. Is immediate to obtain the welfare for this degree of collusion (consumer

---

[7]This result is not general. If economies of scale exist, a merger may increase both concentration and welfare (Farrell and Shapiro, 1990b).

surplus calculated with q_1^c and q_2^c) which is equal to 0.0605. If for instance the cost parameter of the inefficient firm increases to $c_1 = 1.1$, then the endogenous degree of collusion decreases (Proposition 3) and now the solution to $\bar{\delta}_1(\alpha, 1.1) = 0.4$ is $\alpha = 0.6483$. In this case, welfare increases (Proposition 4) and is equal to 0.0606. Finally, it can be obtained that if $\delta = 0.4$, welfare is maximized if $c_1 = 1.06$.

4   Concluding comments

We have developed a theoretical framework to study how firms' cost asymmetry affects the possibility that a collusive agreement can be sustained. Contrary to the usual assumption made in most oligopoly models, it is assumed that a partial tacit collusive agreement can be sustained where firms care about the other firms' profits just to certain extent. We show that even though cost asymmetry hinders collusion, partial collusion can always be sustained regardless of the cost asymmetry. An endogenous degree of collusion also shows, however, that with cost asymmetry there is a limit to the degree of collusion that can be sustained. Another interpretation of our results is that cost asymmetry is not necessarily a restraint for collusion as long as firms are able to sustain the maximum degree of collusion contingent on their discount factor.

Regarding the welfare analysis, the existence of partial collusion decreases consumer surplus. However, we also show that if firms' cost asymmetry increases (for instance due to the increase in the marginal cost of production of a firm), welfare may also be enhanced. This is true because more asymmetry may lead firms to sustain a smaller degree of collusion.

The framework we have worked with is, admittedly, a particular one. To analyze real-world cartels, additional research is required, and for instance a wider range of demand functions should also be considered. It would also be interesting to test if our results are robust to using an optimal punishment —the "stick-and-carrot strategies" proposed by Abreu (1986,1988). We believe that those are subjects for future research.

Acknowledgements

Appendix

Proof of Proposition 1. The equation $\frac{\partial \bar{\delta}_1(\alpha,c_1)}{\partial c_1} = 0$ has only one real root in α when $c_1 > 1$, which is $\alpha = 1 + 2c_1$ that cannot exist in our model since $\alpha < 1$. Hence, evaluating the derivative at a particular level $\frac{\partial \bar{\delta}_1(\alpha,c_1)}{\partial c_1}\Big|_{c_1=1} = \frac{-20\alpha(415+\alpha(163+4\alpha))}{3(-3+\alpha)(40+9\alpha)^2} > 0$ suffices to check that $\frac{\partial \bar{\delta}_1(\alpha,c_1)}{\partial c_1} > 0$. On the other hand, $\frac{\partial \bar{\delta}_1(\alpha,c_1)}{\partial \alpha} = 0$ has also only one real root in α when $c_1 > 1$, which is $\alpha = 1 + 2c_1$ that cannot exist either in our model. Therefore, $\frac{\partial \bar{\delta}_1(\alpha,c_1)}{\partial \alpha}\Big|_{c_1=1} = \frac{1000}{(40+9\alpha)^2} > 0$ and $\frac{\partial \bar{\delta}_1(\alpha,c_1)}{\partial \alpha} > 0$. Then since $\bar{\delta}_1(0,c_1) = 0 \ \forall c_1$ and $\bar{\delta}_1(\alpha,1) = \frac{25\alpha}{40+9\alpha} < 1 \ \forall \alpha \in [0,1]$, we can obtain by continuity of the function $\bar{\delta}_1(\alpha,c_1)$ that there exists $\alpha \equiv \alpha^1$ small enough such that $\bar{\delta}_1(\alpha^1,c_1) \in (0,1)$ for each possible value of $c_1 > 1$.
∎

Proof of Lemma 1. Profits are given by $\Pi_1(q_1^c, q_2^c) = \frac{(-3+\alpha)(-3+\alpha^2-c_1(3+\alpha))}{(7+8c_1-\alpha(2+\alpha))^2}$ and $\Pi_2(q_1^c, q_2^c) = \frac{(1+2c_1-\alpha)(2+4c_1+\alpha-\alpha^2)}{(7+8c_1-\alpha(2+\alpha))^2}$. Then, $\frac{\partial \Pi_2(q_1^c,q_2^c)}{\partial \alpha} = \frac{1+16c_1^2+2c_1(5+\alpha(-16+(9-2\alpha)\alpha))+\alpha(-22+\alpha(18+(-6+\alpha)\alpha))}{(7+8c_1-\alpha(2+\alpha))^3}$. It is tedious but straightforward to show that both the denominator and the numerator of this derivative are positive $\forall \alpha \in (0,1)$, $c_1 > 1$ and consequently $\frac{\partial \Pi_2(q_1^c,q_2^c)}{\partial \alpha} > 0$. On the other hand, $\frac{\partial \Pi_1(q_1^c,q_2^c)}{\partial \alpha} = \frac{3(5+4c_1)-2(6+c_1(13+8c_1))\alpha+12(1+2c_1)\alpha^2-2(4+c_1)\alpha^3+\alpha^4}{(7+8c_1-\alpha(2+\alpha))^3}$. Then, we have $\frac{\partial \Pi_1(q_1^c,q_2^c)}{\partial \alpha}\Big|_{\alpha=0} = \frac{3(5+4c_1)}{(7+8c_1)^3} > 0$ and $\frac{\partial \Pi_1(q_1^c,q_2^c)}{\partial \alpha}\Big|_{\alpha=1} = \frac{-(-1+c_1)}{8(1+2c_1)^2} < 0$. Therefore by continuity there exists a $\alpha \in (0,1)$ where $\frac{\partial \Pi_1(q_1^c,q_2^c)}{\partial \alpha} = 0$. Also, we have
$\frac{\partial^2 \Pi_1(q_1^c,q_2^c)}{\partial^2 \alpha} = \frac{2(3-64c_1^3+105\alpha-102\alpha^2+38\alpha^3-13\alpha^4+\alpha^5-32c_1^2(5-5\alpha+2\alpha^2)-c_1(103-248\alpha+158\alpha^2-64\alpha^3+3\alpha^4))}{(-7-8c_1+\alpha(2+\alpha))^4}$.

The second order condition also holds because the denominator of the last derivative

is negative. This can be easily proved for instance with the standard software Mathematica that shows that there is no root for the equation $\frac{\partial^2 \Pi_1(q_1^c, q_2^c)}{\partial^2 \alpha} = 0$ if $\alpha \in (0, 1)$ and $c_1 > 1$. Then, it is immediate to check, for instance, that

$$\frac{\partial^2 \Pi_1(q_1^c, q_2^c)}{\partial^2 \alpha}\bigg|_{c_1=2} = \frac{2(-5+\alpha)(271+\alpha(-194+\alpha(96+(-14+\alpha)\alpha)))}{(-23+\alpha(2+\alpha))^4} < 0 \blacksquare.$$

Proof of Proposition 2. By definition, if $\delta < \delta^*$ the maximum degree of collusion sustainable for each possible value of δ is always smaller than α^*. In this case, from Lemma 1 we know that profits for firm 1 increase with α if $\alpha < \alpha^*$. Consequently, firm 1 is willing to sustain the maximum possible α. The function $\bar{\delta}_1(\alpha, c_1)$ gives the cutoff of the discount factor needed to sustain α. Then, for a particular value of δ and c_1 (namely $\bar{c}_1$), the solution of $\alpha = \bar{\delta}_1^{-1}(\delta, \bar{c}_1)$ is the degree of collusion willing to be sustained. Note that since $\bar{\delta}_1(\alpha, c_1)$ increases with α, smaller values of α than the one obtained in $\alpha = \bar{\delta}_1^{-1}(\delta, \bar{c}_1)$ can also be sustained for this particular value of δ. But form Lemma 1 we know that firm 1 would obtain lower profits. On the contrary, if $\delta \geq \delta^*$, a degree of collusion larger than α^* can be sustained. However, from Lemma 1, firm 1 is willing to sustain $\alpha = \alpha^*$ since in this case profits are maximized $\blacksquare$.

Proof of Proposition 3. From the proof of Proposition 1 we know that $\frac{\partial \bar{\delta}_1(\alpha, c_1)}{\partial \alpha} > 0$ and $\frac{\partial \bar{\delta}_1(\alpha, c_1)}{\partial c_1} > 0$. The first part of the result follows directly because the endogenous degree of collusion is obtained $\alpha = \bar{\delta}_1^{-1}(\delta, \bar{c}_1)$. For the second part of the result, we have to check that $\frac{\partial \alpha^*}{\partial c_1} < 0$. It is straightforward to obtain the profit function for firm 1 as a function of c_1 and α, $\Pi_1(c_1, \alpha) = \frac{(-3+\alpha)(-3+\alpha^2-c_1(3+\alpha))}{(7+8c_1-\alpha(2+\alpha))^2}$. Then α^* is obtained maximizing $\Pi_1(c_1, \alpha)$ with respect to α and obviously does not depend on δ. The solution unfortunately cannot be explicitly obtained. However, the equation $\frac{\partial \Pi_1(c_1, \alpha)}{\partial \alpha} = 0$ can be solved for c_1 giving

$c_1(\alpha) = \frac{6+12\alpha^2-\alpha^3+\alpha(-13+\sqrt{\frac{(-3+\alpha)^2(4+12\alpha+21\alpha^2-2\alpha^3+\alpha^4)}{\alpha^2}})}{16\alpha}$. Then, $\alpha^* = c_1^{-1}(c_1)$. It is easy to check

that $\frac{\partial c_1(\alpha)}{\partial \alpha} < 0$. Hence, using Lagrange's notation, the derivative of the inverse function is given by $\frac{\partial \alpha^*}{\partial c_1} = \frac{1}{\frac{\partial c_1(\alpha)}{\partial \alpha}\big|_{c_1^{-1}(c_1)}} < 0$. In words, since $c_1(\alpha)$ is decreasing so is also its inverse function $\blacksquare$.

Proof of Proposition 4. The welfare function that we denote by $W(c_1, \alpha)$ in the present case is given by $\frac{Q^2}{2}$. It is easy to check that $W(c_1, \alpha) = \frac{2(2+c_1-\alpha)^2}{(7+8c_1-\alpha(2+\alpha))^2}$. Consequently, if c_1 increases we have two different forces that go on opposite directions. On the one hand

$\frac{\partial W(c_1,\alpha)}{\partial c_1} = -\frac{4(2+c_1-\alpha)(-3+\alpha)^2}{(7+8c_1-\alpha(2+\alpha))^3} < 0$ thus welfare decreases. On the other hand, from the proof of Proposition 1 we know that $\frac{\partial \bar{\delta}_1(\alpha,c_1)}{\partial c_1} > 0$ which implies that the endogenous degree of collusion decreases (hence if all else equal welfare increases). We basically have to prove that when $c_1 \in (1,\infty)$ one force is not always larger than the other. This would be equivalent to check that the function $W(c_1,\alpha)$ evaluated at the endogenous degree of collusion obtained in Proposition 2 ($\alpha = \bar{\delta}_1^{-1}(\delta,\bar{c}_1)$ if $\delta < \delta^*$ and $\alpha = \alpha^*$ if $\delta \geq \delta^*$) has a maximum with respect to c_1. The function obtained is unfortunately intractable.

We can prove the result in a different way. We have two different cases: i) if $\delta < \delta^*$ and ii) if $\delta \geq \delta^*$. i) We denote the endogenous value of α (as a function of c_1) like $\alpha(c_1)$. Applying the chain rule we have that $\frac{\partial W(c_1,\alpha(c_1))}{\partial c_1} = \frac{4(2+c_1-\alpha(c_1))(-3+\alpha(c_1))(3-\alpha(c_1)+(1+2c_1-\alpha(c_1))\alpha'(c_1))}{(7+8c_1-\alpha(c_1)(2+\alpha(c_1)))^3}$ where $\alpha'(c_1) \equiv \frac{\partial \alpha(c_1)}{\partial c_1}$. We know from Proposition 3 that $\alpha'(c_1) < 0$. On the other hand, obviously $\alpha(c_1) \in (0,1)$. Then, c_1 such that $\frac{\partial W(c_1,\alpha(c_1))}{\partial c_1} = 0$ exists if $3 - \alpha(c_1) + \alpha'(c_1)(1+2c_1 - \alpha(c_1)) = 0$. We have that $-\alpha(c_1) + \alpha'(c_1)(1+2c_1-\alpha(c_1)) < 0$ and the critical point is obtained when $-\alpha(c_1) + \alpha'(c_1)(1+2c_1-\alpha(c_1)) = -3$. We have to check the range of $\alpha'(c_1)$. Since $\alpha(c_1) = \bar{\delta}_1^{-1}(\delta, c_1)$ and it can be easily checked that the range of $\frac{\partial \bar{\delta}_1(\alpha,c_1)}{\partial c_1}$ varies between 0 (when c_1 tends to 1 and α tends to 0) and $\frac{4}{3}$ (when c_1 tends to ∞ and α tends to 1), consequently the range of $\alpha'(c_1)$ varies (in absolute value) from $\frac{3}{4}$ to ∞. This implies that a critical point in the maximization of $W(c_1,\alpha(c_1))$ with respect to c_1 can always be obtained for a finite c_1 and $\alpha \in (0,1)$. The only thing left then is to check that this critical point is a global maximum and not a minimum. This is true since $W(c_1,\alpha)$ monotonically decreases with c_1 and the minimum is obtained by calculating $\lim_{c_1 \to \infty} W(c_1,\alpha) = \frac{1}{32}$. ii) In this case $\alpha(c_1) = \alpha^*$. From the proof of Proposition 3 we know that $\frac{\partial \alpha^*}{\partial c_1} < 0$, now we have to check that this derivative range varies in absolute value from 0 to ∞. We know that the equation $\frac{\partial \Pi_1(c_1,\alpha)}{\partial \alpha} = 0$ gives an explicit solution on $c_1(\alpha)$ where $\alpha^* = c_1^{-1}(c_1)$. This univariate function obtained (see proof of Proposition 3) with domain between 0 and 1 has a range between $-\infty$ and 0. As a consequence, since $\frac{\partial \alpha^*}{\partial c_1} = \frac{1}{\frac{\partial c_1(\alpha)}{\partial \alpha}\Big|_{c_1^{-1}(c_1)}}$, the range of the inverse function also varies between $-\infty$ and 0 and a solution to $\frac{\partial W(c_1,\alpha(c_1))}{\partial c_1} = 0$ always exists ∎.

Proof of Corollary 1. Straightforward.

References

-Abreu, D. (1986). "Extremal Equilibria of Oligopolistic Supergames". *Journal of Economic Theory 39 (1), 191-225.*

-Abreu, D. (1988). "On the Theory of Infinitely Repeated Games with Discounting". *Econometrica 56, No.2, 383-396.*

-Bain, J.S. (1948). "Output quotas in imperfect cartels". *Quarterly Journal of Economics 62, 617-622.*

-Ciarreta, A., Gutiérrez-Hita, C. (2011). "Collusive behaviour under costs asymmetries when firms compete in supply functions". *Mimeo.*

-Donsimoni, M.P., Economides, N.S., Polemarchakis, H.M. (1986). "Stable Cartels". *International Economic Review 27(2), 317-327.*

-Eckbo, Paul L. (1976). The Future of World Oil. *Cambridge: Ballinger Publishing.*

-Escrihuela-Villar, M. (2008). "Partial coordination and mergers among quantity-setting firms". *International Journal of Industrial Organization 26, 803–810.*

-Farrell, J., Shapiro, C. (1990a). "Asset ownership and market structure in oligopoly". *RAND Journal of Economics 21, 275-292.*

-Farrell, J., Shapiro, C. (1990b). "Horizontal Mergers: An Equilibrium Analysis". *The American Economic Review 80 (1), 109-126.*

-Friedman, J.W., (1971). "A Non-cooperative Equilibrium for Supergames". *Review of Economic Studies 28, 1-12.*

-Fundenberg, D. and Maskin, E. (1986). "The folk theorem in repeated games with discounting or with incomplete information". *Econometrica 54, 533-554.*

-Griffin, James M. (1989). "Previous Cartel Experience: Any Lesson for OPEC?" in Lawrence R. Klein and Jamie Marquez, eds. *Economics in Theory and Practice: An Eclectic Approach. Dordrecht: Kluwer Academic Publishers, pp. 179-206.*

-Harrington, J. (1991). "The determination of price and output quotas in a heterogeneous cartel". *International Economic Review 32, 767-792.*

-Levenstein, M. and Suslow, V. Y. (2006). "What Determines Cartel Success?". *Journal of Economic Literature, 44(1), 43-95.*

-Mason, C.F., Phillips, O.R. and Nowell, C. (1992), "Duopoly Behavior in Asymmetric Markets: an Experimental Evaluation". *Review of Economics and Statistics 74, 662-670.*

-Osborne, M.J. and Pitchik C. (1983). "Profit-sharing in a collusive industry". *European Economic Review 22, 59-74.*

-Patinkin, D. (1947). "Multiple-plant firms, cartels, and imperfect competition". *Quarterly Journal of Economics 61, 173-205.*

-Pittman, Russell W. (2007). "Consumer Surplus as the Appropriate Standard for Antitrust Enforcement". *Competition Policy International, 3(2).*

-Reynolds, R. and Snapp, B. (1986). "The competitive effects of partial equity interests and joint ventures". *International Journal of Industrial Organization 4, 141-153.*

-Rothschild, R. (1999). "Cartel stability when costs are heterogeneous". *International Journal of Industrial Organization 17, 717-734.*

-Schmalansee, R. (1987). "Competitive advantage and collusive optima". *International Journal of Industrial Organization 5, 351-367.*

-Vasconcellos, H. (2005). "Tacit Collusion, Cost Asymmetries, and Mergers". *RAND Journal of Economics 36, 39-62.*

-Verboven, F. (1997). "Collusive behavior with heterogeneous firms". *Journal of Economic Behavior and Organization 33, 121-136.*