Directed Search with Endogenous Search Effort.[*]

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Abstract

In this paper I develop an urn-ball game type matching model with ex-ante identical agents and firms. This is a directed search market structure. Agents can freely choose the amount of applications they make. Firms post vacancies, receive applications and make a single take-it-or-leave-it wage offer to a randomly chosen applicant. As agents make multiple applications, firms compete for the particular applicant as in a first-price sealed-bid auction with an unknown number of biders. Miscoordiantion and asymmetry of information induce a mixed strategy by firms. They make wage offers as extractions from a continuous distribution, inducing wage dispersion among agents. The particular shape of the obtained wage distribution depends on the search effort of agents. The expected wage can be seen as an endogenously determined bargaining power. The particular wage that an agent receives is stochastic, a combination of luck and effort. The miscoordination in search effort produces a negative externality on the agents side. The relation between search effort and labor market tightness(understood as the vacancy/worker ratio) is not monotonic. It is positive for low values of market tightness and negative for high values.

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JEL classification: J31, J65, D83

1   Introduction

One of the frictions present in labor markets is the matching process, a mechanism required to match firms and workers. In most of the literature, the matching mechanism is just an exogenously imposed function describing the relation between market tightness and the number of matches. In most of these models, the search direction is not addressed and search effort is absent. To address the search direction a description of the market structure is required. The urn-ball game induces such a market structure. In this set-up one of the sides of the market, typically the worker, initiates the process by making an initial contact with the other side, typically the firm. However, in the standard urn-ball game, search effort is not present since agents make a single application. Albrecht, Gautier and Vroman (2006, hereafter AGV) solve this question by allowing agents to make any number of applications. However, in their work the particular number of applications is an exogenous variable.[1] The wage bargaining process, absent in the urn-ball game, becomes relevant. Since agents make more that one application they can receive more than one offer. What AGV propose is a wage posting set up where, in the case that more than one firm chose the same applicant, they engage in a Bertrand competition for that particular applicant. Then, in equilibrium, firms post the reservation wage. An agent with only one offer gets the reservation wage, since it does not induce Bertrand competition among firms, while an agent with more than one offer gets the whole production value as a result of Bertrand competition. The result is a wage dispersion concentrated in two extreme cases. It also assumes that firms have information about the number of offers that agents receive. It implies losses for all firms that make a match after a Bertrand competition.

This paper has two main differences with the work by AVG. The wage bargaining mechanism and the explicit endogeneity of search effort. I assume that firms do not post wages, only vacancies. After receiving applications, firms make a single, take-it-or-leave-it, wage offer to a randomly chosen applicant. Firms compete to be

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[1] AGV endogenaizes the search effort in section 5.1 as a robustness check. What they do is to look for a set of equilibrium values that sustain a particular equilibrium value of applications. Here I will solve directly the behavior of agents.

the highest offer that the agent receives, without information about the number of offers received by the applicant. This turns out to be identical to a sealed first-price auction with an unknown number of bidders. The resulting wage offer distribution is continuous between the reservation wage and an upper-bound value that is strictly lower that the total value of production. This implies that a match yields always positive profits. The shape of the wage distribution will depend both on labor market tightness and on search effort. Intuitively, the expected wage can be seen as an endogenously determined bargaining power that depends on the search effort of agents, while the highest offer that the agent receives will be an extraction from this distribution. The particular wage that the agent obtains is, therefore, a combination of luck and effort. Effort induces the shape of the distribution while luck determines the particular value of the extracted wage.

The relation obtained between market tightness and search effort is not monotonic. It is positive for low values of market tightness and negative for high values. This result implies that an increase in the number of existing job vacancies induces an increase(decrease) in the search effort when the number of vacancies is low(high).

In the model there are severe problems of coordination and asymmetry of information. These problems will produce negative externalities. The most tractable externality is related to the miscoordination of agents regarding the number of applications they send. The equilibrium result shows that agents send too many applications. The excessive amount of applications produces a negative externality on the expected return of agents, due to excessive competition. A model where agents can coordinate in the number of applications is easy to implement and, therefore, the value of the negative externality can be measured comparing both results.

The model accepts a wide set of extensions, as introducing heterogeneity in one (or both) sides of the market or additional wage offer rounds.

The rest of the paper is organized as follows. In section 2, I solve the static model as a one shoot game. Section 3 present the results relative to the externality produced by the miscoordination of agents. Section 4 presents a robustness check including the free entry condition for firms and a dynamic version of the model.

Section 5 presents a set of extensions of the static model and section 6 concludes.

2 Model

I consider an economy where N agents are looking for a job and there are V posted job vacancies. Both agents and vacancies are homogeneous and both numbers are common knowledge. First firms post vacancies, one per firm. Agents observe the number of vacancies V and send an amount S of applications, I assume that they cannot apply twice to the same vacancy. Each application has a cost c for the agent. Once applications are made, firms with at least one application choose one, at random, and make a wage offer W. The value of W and the identity of the chosen applicant are private information. Agents collect offers and accept the highest one, provided that it exceeds the reservation wage $\underline{w}$.[2] An accepted offer makes a match and the vacancy is fulfilled. Each fulfilled vacancy generates a value Q for the firm.

There are several coordination problems involved. Agents cannot coordinate in which firms they make applications, neither in the number of applications. Firms cannot coordinate in which agents they make offers or in the amount of the offer. There is asymmetry of information as firms do not know the number of offers that an agent has received, neither the value of particular offers.

I will use a symmetric equilibrium concept where homogeneous players behave identically. First I will solve all the probabilities involved in the matching process and then I will solve the model backwards. Then I will solve the wage offer decision of firms and, finally, the number of applications that agents wish to make.

2.1 Probability construction

In equilibrium all agents will make the same number S of applications. Since an agent sends S applications, a firm has a probability $\frac{S}{V}$ of receiving an application from a given agent and a probability $\left(1 - \frac{S}{V}\right)$ of not receiving it. It will not have any application with probability $\left(1 - \frac{S}{V}\right)^N$. A firm receives at least one application with

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[2]Or randomizes between the highest offers if there is a tie.

probability $1-\left(1-\frac{S}{V}\right)^{N}$. The total number of offers made in the economy is equal to the number of firms that receive at least one application, that is $V\left(1-\left(1-\frac{S}{V}\right)^{N}\right)$. The probability that a particular application is successful is:

$$O(S,V,N) = \frac{V\left(1-\left(1-\frac{S}{V}\right)^{N}\right)}{SN}. \tag{1}$$

For N large enough this expression can be approximated by:

$$O(S,\theta) = \frac{\theta\left(1-e^{-S/\theta}\right)}{S}, \tag{2}$$

where $\theta = \frac{V}{N}$ describes the labor market tightness.

An agent making S applications has a probability $1-(1-O(S,\theta))^{S}$ of receiving at least one wage offer. Each worker with at least one wage offer makes a match. The total number of matches is:[3]

$$m(S,\theta) = N(1-(1-O(S,\theta))^{S}). \tag{3}$$

2.2 Firms wage offer

Each firm makes a single wage offer to a randomly chosen application. Firms will anticipate the number of applications made, and the behavior of the rest of the firms. The firm makes the wage offer that yields the highest expected profit, or randomize between the wages that yield the highest expected profit. The firm chooses its offer among the set of wages Ω_{W} that yield the highest expected profit:

$$\Omega_{W} = \arg\max\left\{(Q-W)\,F(W)\right\}, \tag{4}$$

where $(Q-W)$ is the profit if the wage is W and $F(W)$ is the probability that a wage offer of W is accepted.

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[3]This result anticipates that an offer is always higher than the reservation wage.

The agents accepts a particular offer only if it is the highest one among all received offers. This implies that $F(W)$ is equivalent to the cumulative distribution function of the highest offer received from other firms. The firm does not know the exact number of offers that the agent has received, but it can construct $F(W)$ in order to compute the expected profit of a particular wage offer. Firms choose the wage offer in an, ex-ante, identical way. To be as general as possible I assume that the strategy space of firms is $B(W)$, where $B(W)$ represents all possible cumulative distribution functions over Ω_W.

Then $F(W)$ can be constructed as follows:

$$F(w \leq W) = (1 - O(S, \theta))^{S-1} + \sum_{i=1}^{S-1} \binom{S-1}{i} O(S, \theta)^i (1 - O(S, \theta))^{S-1-i} B(w \leq W)^i, \tag{5}$$

if $B(W)$ is continuous in W, or

$$F(w \leq W) = F(w < W) + \sum_{i=1}^{S-1} \tfrac{1}{i} \binom{S-1}{i} O(S, \theta)^i (1 - O(S, \theta))^{S-1-i} B(w = W)^i, \tag{6}$$

if $B(W)$ has a discontinuity in W.

This is identical to a sealed bid first-price auction with an unknown number of bidders, where all bidders value the good exactly the same.[4]

Next I describe the behavior of firms in a set of lemmas.

Lemma 2.1 *Any wage offer must be higher or equal than the reservation wage $\underline{w}$ and lower or equal than the production value Q.*

Proof. Any offer lower than the reservation wage yields negative profits as the probability of acceptance is zero. Then it is dominated by the reservation wage. Any offer higher than the production value yields negative profit so it is also dominated. ∎

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[4]The exact number of bidders is unknown, but it is bounded by S.

Lemma 2.2 *The distribution $B(W)$ cannot have any discontinuity and, therefore, $F(W)$ has a unique discontinuity at the reservation wage, due to the probability that an agent has no other offer.*

Proof. If $B(W)$ has a discontinuity at some wage, this wage offer does not belong to Ω_W. An epsilon higher offer yields a higher expected profit since there is a discontinuity in $F(W)$ that increases drastically the probability of acceptance. ■

The above results imply that $F(W)$ can be stated as

$$F(W \leq w) = (1 - O(S, \theta))^{S-1} + \sum_{i=1}^{S-1} \binom{S-1}{i} O(S,\theta)^i (1 - O(S,\theta))^{S-1-i} B(W \leq w)^i, \tag{7}$$

that is equivalent to

$$F(W) = ((1 - O(S, \theta)) + O(S, \theta) B(W))^{S-1}. \tag{8}$$

It also implies that the probability that a firm makes a particular wage offer is zero, that is, $B(w = W) = 0$.

Lemma 2.3 *The lowest offer is the reservation wage $\underline{w}$ and the probability of acceptance is equal to the probability of having no other offer.*

Proof. If the reservation wage were not the lowest offer, the probability of acceptance of the lowest offer and the reservation wage would be the same. Then, to offer the reservation wage $\underline{w}$ yields a higher profit. As $B(w = \underline{w}) = 0$, the reservation wage is accepted only if there is no other offer. ■

Lemma 2.3 implies that:

$$F(\underline{w}) = (1 - O(S, \theta))^{S-1}. \tag{9}$$

Lemma 2.4 *The highest offered wage $\overline{w}$ must be equal to $Q - (Q - \underline{w})(1 - O(S, \theta))^{S-1}$.*

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Proof. The expected profit is constant for all wages in the set Ω_W. The highest wage has probability one of acceptance and yields a profit $(Q - \overline{w})$. This profit must be equal to $(Q - \underline{w})(1 - O(S, \theta))^{S-1}$, the expected profit of offering the reservation wage $\underline{w}$. ∎

Lemma 2.5 *If all firms use the same distribution of wage offers, then the domain of $B(W)$ must coincide with the domain of Ω_W, this domain must be connected, compact and must contain more than one value.*

Proof. Firms make offers taking wages from the set Ω_W. A priori they can make offers from different subsets since they are indifferent among wages in Ω_W. The first part of the lemma claims that all firms must attach some probability to all wages in Ω_W. This is due to the construction of Ω_W that depends on $F(W)$ that, in turn, depends on $B(W)$. Observe that if Ω_W contains more than one value it must hold that $(Q - W)F(W)$ is identical in all of them. Since $F(W)$ is a function without discontinuities above $\underline{w}$, then Ω_W must be connected. If there were two different subsets, any intermediate value will dominate them because it will give higher expected profit.[5] Then $F(W)$ must be strictly increasing for all values in Ω_W and, therefore, $B(W)$ must also be strictly increasing for all those values. The set is bounded because the wage offers cannot be lower than $\underline{w}$ and cannot exceed $\overline{w}$, and it is closed because both values are in the set. The set is connected, bounded and closed so it is compact. The set cannot be a singleton because offering an epsilon higher wage will yield a higher profit. ∎

Lemma 2.5 implies that Ω_W is a connected set where:

$$(Q - W_i) F(W_i) = (Q - W_j)F(W_j) \quad \forall W_i, W_j \in \Omega_W. \tag{10}$$

Since $\underline{w} \in \Omega_W$ and $F(\underline{w}) = (1 - O(S, \theta))^{S-1}$ then:

$$F(W) = \frac{Q - \underline{w}}{Q - W}(1 - O(S, \theta))^{S-1} \text{ for } W \in [\underline{w}, \overline{w}]. \tag{11}$$

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[5]It has the same probability to be accepted than the lowest wage in the upper set that belongs to Ω_w but yields a higher profit.

With this equation and using Equation(8), $B(W)$ can be written as:

$$B(W) = \frac{(1 - O(S, \theta))}{O(S, \theta)} \left(\left(\frac{Q - \underline{w}}{Q - W} \right)^{\frac{1}{S-1}} - 1 \right). \tag{12}$$

The behavior of firms is characterized by a continuous cumulative distribution function over a connected set of wages. Note that this implies wage dispersion among identical agents that make an identical number of applications, this dispersion does not require any exogenous random variable. The wage distribution is a result of the competition among firms to hire a particular worker and of their lack of information. The number of offers received by the worker is not known by firms, so they will attach a positive probability to the event that they are making the only offer. To offer the reservation wage has a positive expected profit and all other offers must yield the same expected profit. The competition among firms is assumed to be restricted to a single take-it-or-leave-it offer. This assumption is sustained by the same lack of information. A firm will not enter in to ex-post competition after a rejection, because it would be optimal for the agent to reject any offer in order to induce a wage increase.[6]

The firms cumulative wage offer distribution, for $Q = 1$, $\theta = 1$ and $\underline{w} = 0$, has the following shape:

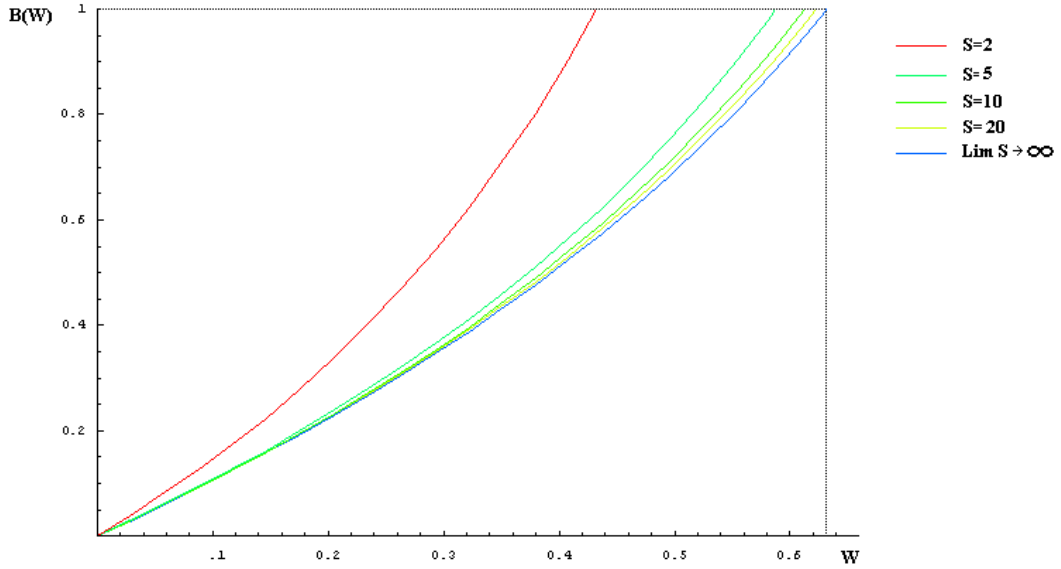[6] Any offer lower than the value of total production.

Figure 1: CDF of wage offers.

Firms react to the number of applications that agents send. If agents send a single application, firms will not have incentives to offer more than the reservation wage. The increase in the number of applications rises the probability that an agent has more that one offer. This drives the wage offer distribution closer to its limiting value. An increase in the number of applications moves to the right the upper bound of the distribution. The distribution related to a particular number of applications stochastically dominates all distributions corresponding to fewer applications.

The wage offer distribution converges to the limiting distribution:

$$\lim_{S \to \infty} B(W) = \frac{1}{\theta} \ln \left(\frac{Q - \underline{w}}{Q - W} \right) \text{ for } W \in [\underline{w}, \overline{w}] \tag{13}$$

If the agents send an infinite number of applications the probability of receiving a particular number of applications can be expressed as a Poison distribution. This approach is followed by Halko, Kultti and Virrankoski (2008). Here I am interested in the problem related to the choice of search effort. Agents will not make an infinite number of applications.

2.3 Search effort

Agents maximize the expected return corresponding to the number of applications S. They take into account the cost of applications and the behavior of the rest of the society. Agents observe the market tightness θ and give their best response to a scenario where the rest of agents make a fixed number of applications $\overline{S}$.

Firms offer wages according to $B(W)$ that is related to $\overline{S}$. Agents are concerned about the highest offer they receive, since this will be the offer they will accept. Agents then compute $H(W)$, the cumulative distribution function of the highest offer received, that can be constructed as:

$$H(w \le W) = \sum_{i=1}^{S} \binom{S}{i} O(\overline{S}, \theta)^i (1 - O(\overline{S}, \theta))^{S-i} B(w \le W)^i, \tag{14}$$

that is equivalent to:

$$H(W) = \left((1 - O(\overline{S}, \theta)) + O(\overline{S}, \theta) B(W)\right)^S - (1 - O(\overline{S}, \theta))^S. \tag{15}$$

However this is not the wage of agents yet. Agents have an outside option. If they do not receive any offer, they earn the reservation wage. The cumulative distribution function of the wage associated to S applications is:[7]

$$R(W) = \left((1 - O(\overline{S}, \theta)) + O(\overline{S}, \theta) B(W)\right)^S \text{ for } W \in [\underline{w}, \overline{w}]. \tag{16}$$

Plugging $B(W)$ in this function I get:

$$R(W) = (1 - O(\overline{S}, \theta))^S \left(\frac{Q - \underline{w}}{Q - W}\right)^{\frac{S}{\overline{S}-1}}. \tag{17}$$

The distribution of the wage that agents perceive, for fixed $\overline{S} = 20$, $\theta = 1$, $Q = 1$, $\underline{w} = 0$, is:

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[7]The return takes into account that the agent will receive the reservation wage if there are no offers.
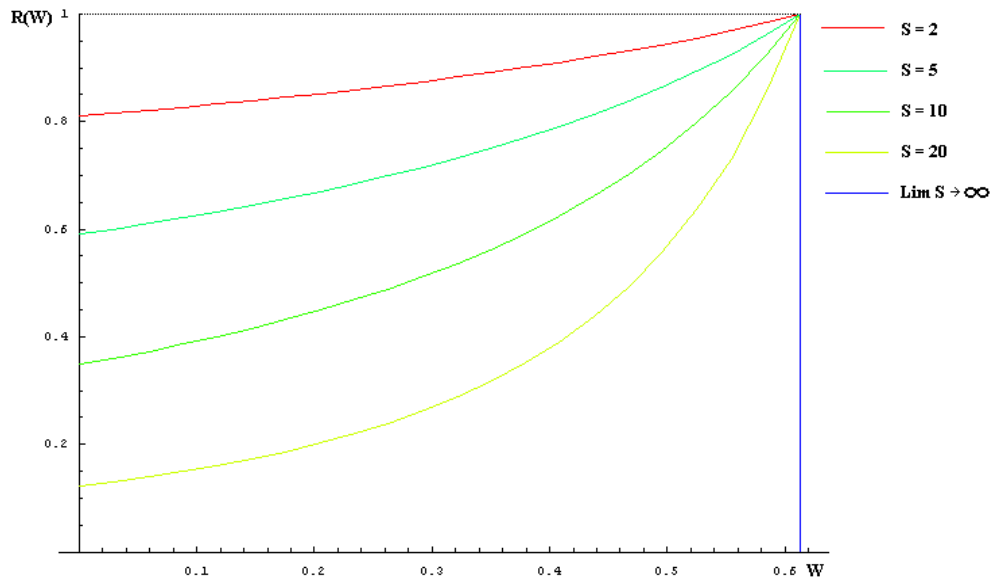
Figure 2: Wage, taking as given the behaviour of the rest of the society.

Here agents do not take into account the relation between their behavior and the behavior of the rest of the agents, they just take $\overline{S}$ as fixed. As in equilibrium all agent make S applications, the equilibrium cumulative distribution of the return looks like follows:
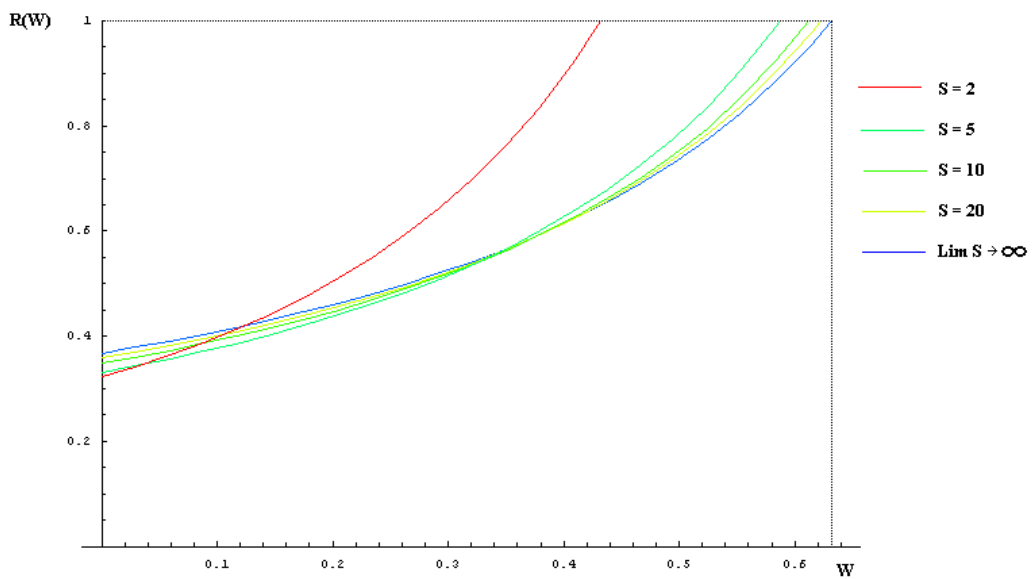

Figure 3: Wage distribution.

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when agents take $\overline{S}$ as given, they believe that by increasing the number of applications they will, individually, face a distribution that stochastically dominates the distributions corresponding to fewer applications. In equilibrium the distributions cross each other, so there is no dominance. This is reflected in the negative externality related to the excessive number of applications in equilibrium.

Agents choose S to maximize the expected return, taking as given the behavior of the rest of the society.

The density function of the wage is:

$$r(W) = \frac{S}{\overline{S}-1}\left(1 - O(\overline{S},\theta)\right)^S \left(\frac{Q-\underline{w}}{Q-W}\right)^{\frac{S}{\overline{S}-1}}\frac{1}{Q-W}. \tag{18}$$

The problem of the agents is to maximize the return, that is, the expected wage minus the cost of applications:

$$\underset{S}{Max}\int_{\underline{w}}^{Q-(Q-\underline{w})(1-O(\overline{S},\theta))^{\overline{S}-1}} Wr(W)dW - Sc. \tag{19}$$

This can be written as:

$$\underset{S}{Max}\ \left(Q - \frac{S(Q-\underline{w})\left(1-O(\overline{S},\theta)\right)^{\overline{S}-1} - \left((\overline{S}-1)Q-S\underline{w}\right)\left(1-O(\overline{S},\theta)\right)^S}{1+S-\overline{S}}\right) - Sc. \tag{20}$$

This function has a unique maximum with respect to S.

The first-order condition, once I impose the symmetry condition $\left(S = \overline{S}\right)$ is:

$$(S-1)\left(1-O(S,\theta)\right)^S \left(\frac{(Q-\underline{w})O(S,\theta)}{1-O(S,\theta)} - \left(Q-\underline{w}\frac{S}{S-1}\right)\left(\ln(1-O(S,\theta))^{-1}\right)\right) = c. \tag{21}$$

Observe that in this expression, the term $\left(Q-\underline{w}\frac{S}{S-1}\right)$ makes the reservation wage more relevant than what it is generally assumed. It is usual to assume that the relevant variable is just the difference between production and the reservation wage, so the reservation wage is set to zero. This simplification cannot be done here

unless the number of applications goes to infinity. Still I can express the relevant variables as shares of the production, by setting $Q = 1.$[8]

Unfortunately, Expression21 , cannot be solved analytically, but it can be easily solved using a computer.

Fixing $Q = 1$, $\underline{w} = 0$ and $c_a = 0.001$, the optimal number of applications as a function of θ, has the following shape:
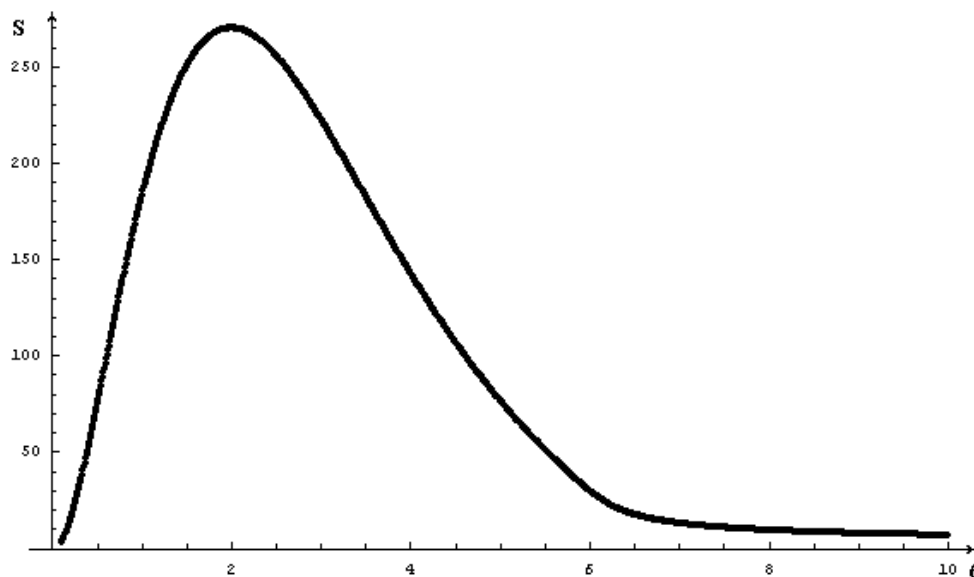

Figure 4: Optimal number of applications as a function of θ.

The optimal search effort has not a monotonic relation with market tightness. For low values of market tightness the search effort is increasing with θ while for high values it is decreasing. This implies a procyclical behavior in the low range of θ and a countercyclical behavior for high rates of θ. In this particular example the procyclical behavior is found for the most realistic values of θ. For those values, a decrease in θ, due to a decrease in the number of vacancies or to an increase in the number of agents, will decrease the search effort. In this example, the countercyclical behavior is found for very high values of θ, where the number of vacancies is at least twice as high as the number of agents. Even when this seems unrealistic, this pattern

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[8]Many other variables, that are not considered in the model, can be easily included as changes in the existing set of variables. For example a linear wage tax can be included as a change the reservation wage to $\left(\frac{w}{1-t}\right)$.

can be found in some sectors. Mostly in sectors related to seasonal activities or with low return unskilled activities.

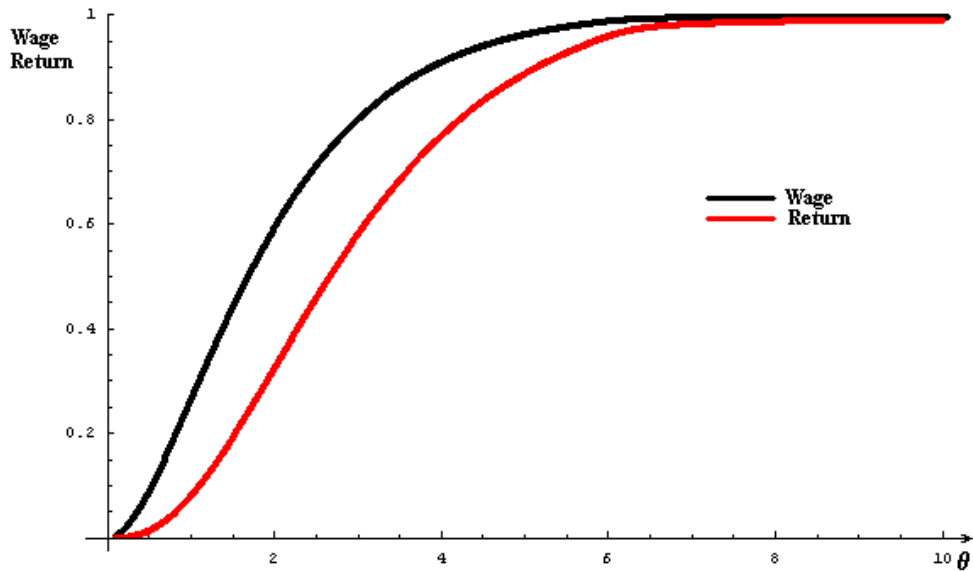The expected return and wage, as a function of θ, are

Figure 5: Expected wage and return.

Here the expected wage is equivalent to the share of total production that agents expect to receive. This can be interpreted as an endogenous measure of the bargaining power of agents that depends on the market tightness θ. Changes in the market tightness will imply different bargaining powers. An increase in the number of agents looking for a job, for example an increase of unemployment, will harm the bargaining power of agents as it implies a decrease in θ.

The particular wage that agents will receive is still stochastic, being the wage cumulative distribution function different for each value of θ. For $\theta = 1$ it has the following shape:
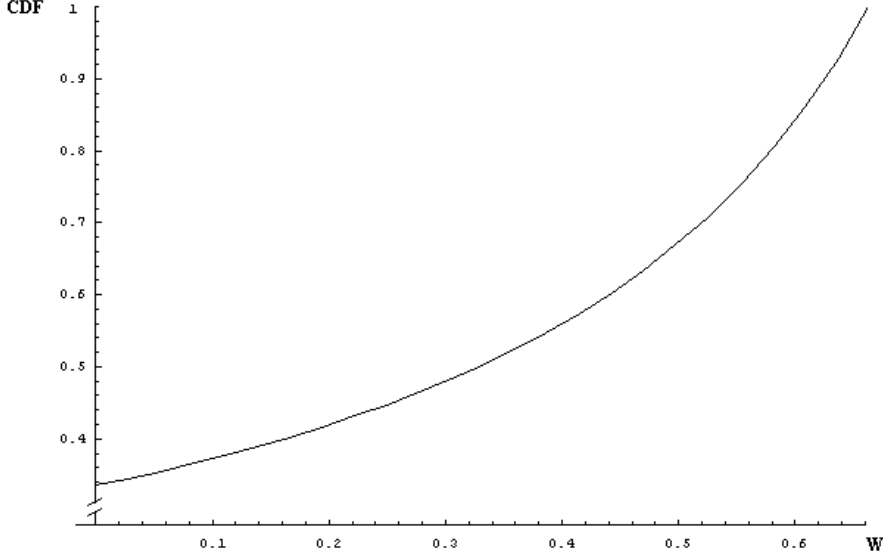
Figure 6: Wage distribution.

This distribution describes how production is shared between firms and agents in each match. Agents wage is extracted from that distribution. Both wage and return are then a question of effort and luck. Effort is related to the number of applications made, that induces the shape of the distribution, and luck to the particular realization of the highest offer. A wage under(above) the expected value, can be understood as bad(good) luck. In this example the expected offer is 0.264.

3   Externality arising from agents miscoordination

The choice of the number of applications by the agents generates an externality. They take as fixed the behavior of the other agents when they choose S. If agents were allowed to coordinate in the number of applications they would solve:

$$\underset{S}{Max} \int_{\underline{w}}^{Q-(Q-\underline{w})(1-O(S,\theta))^{S-1}} Wr(W)dW - Sc, \qquad (22)$$

where $B(W)$ and the upper bound are now functions of S instead of $\overline{S}$.

That expression can be written as:

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$$\underset{S}{Max}\left(1 - (1 - O(S,\theta))^S - SO(S,\theta)(1 - O(S,\theta))^{S-1}\right)Q \qquad (23)$$
$$+ SO(S,\theta)(1 - O(S,\theta))^{S-1}\underline{w} - Sc.$$

This expression is easy to read. Agents maximize the probability of receiving at least two offers times total production, plus the probability of having exactly one offer times the reservation wage, minus the cost of making applications. This is close to the result in AGV, where they assume Bertrand competition among firms. Agents obtain the full value if they have at least two offers and the reservation wage if they receive just one offer.

The first-order condition states that:

$$(a)^{S-1}\left((a)\ln(a)\frac{\partial O(S,\theta)}{\partial S}Q - (1-a)\left(1 + S\frac{\partial O(S,\theta)}{\partial S}\left(\frac{1}{1-a} - \ln a\right)\right)(Q - \underline{w})\right) = c, \quad (24)$$

where $a = 1 - O(S,\theta)$.

The optimal number of coordinated applications, as a function of θ and for the same parameter values than above, is:
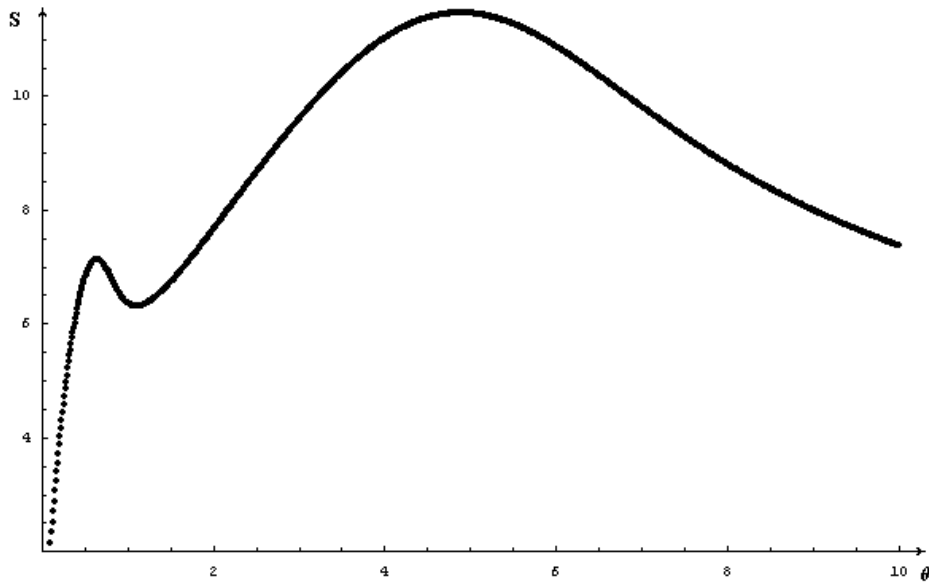

Figure 7: Optimal number of applications if agents can coordinate.
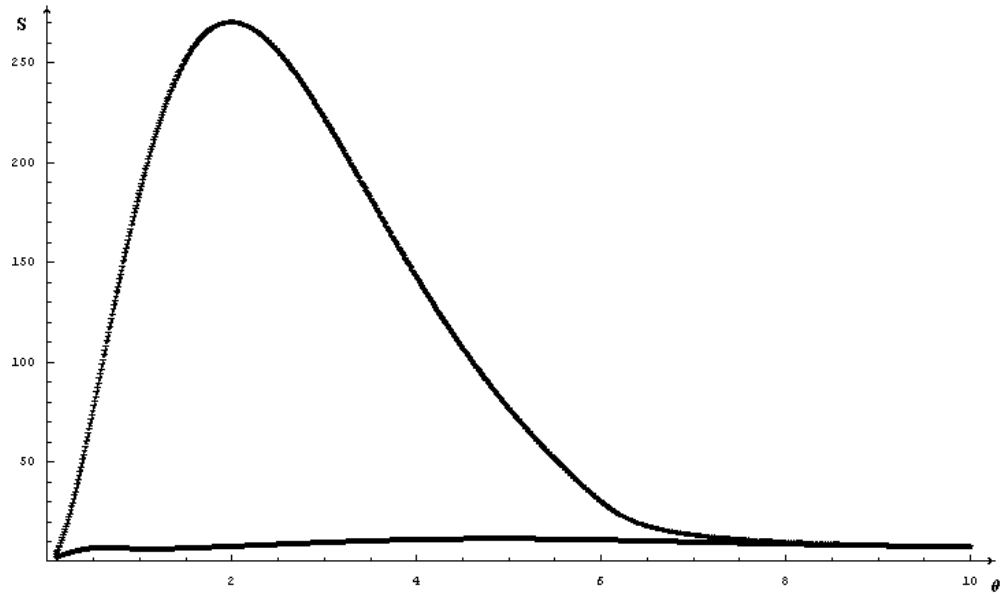
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Plotting both optimal number of applications:

Figure 8: Comparison of optimal number of applications as function of θ.

Here, the upper line corresponds to the scenario without coordination. Clearly agents make send many applications, generating a negative externality. The higher number of applications increases the total cost of applications and reduces the probability that a particular application is accepted.

The value of the negative externality can be obtained comparing the expected revenue in both cases.
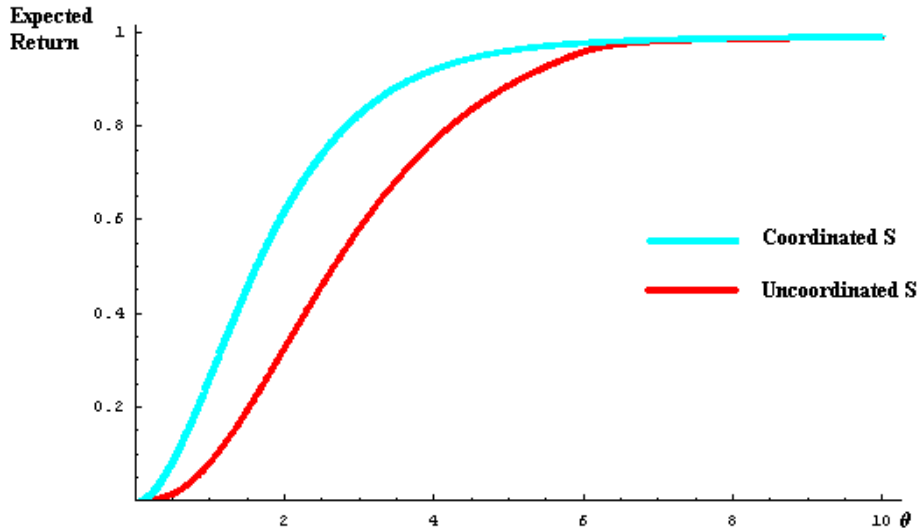
Figure 9: Comparison of expected returns.

The externality is clear and highly relevant.

4   Robustness check

4.1   Market tightness under free entry

Market tightness can be endogeneized. To do so I allow firms to decide if they wish or not to post a vacancy under a free entry condition. Firms post vacancies until the expected profit of opening a vacancy coincides with the cost (k). The expected profit of a firm that receives at least one application is expressed as:

$$(Q - W)\, F(W). \tag{25}$$

This is the profit related to the offered wage times the probability of being accepted. This is a constant value equal to the expected profit of the reservation wage. The expected profit of a firm that receives at least one application is:

$$(Q - \underline{w})\, (1 - O(S, \theta))^{S-1}. \tag{26}$$

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Not all firms receive applications. A firm will receive at least one applications with probability $\left(1 - e^{-S/\theta}\right)$.

The free entry condition can be expressed as:

$$(Q - \underline{w}) \left(1 - O(S, \theta)\right)^{S-1} \left(1 - e^{-S/\theta}\right) = k. \qquad (27)$$

Since N is a fixed exogenous variable and firms choose V, they are in fact choosing θ.

The free entry condition can be rewritten as:

$$(Q - \underline{w})S \left(1 - O(S, \theta)\right)^{S-1} O(S, \theta)N = Vk. \qquad (28)$$

The number of agents that will accept the minimum wage times the associated profit must be equal to the total cost of the open job vacancies. This is similar to the result obtained in AGV.

The free entry equilibrium condition converges, for low values of S, to a limit condition.[9]

$$\theta = -\ln\left(\frac{k}{Q - \underline{w}}\right). \qquad (29)$$

A relatively low value of applications is enough to drive the market tightness close to the limit value that does not depend on S or c. Then, $\underline{w}$ and k can be fixed to support any value of θ. Observe that the reservation wage of agents is again relevant.

4.1.1 Free entry equilibrium

The system composed of Equation(21) and Equation(27) defines the free entry equilibrium.

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[9]Observe that necessarily $k < (Q - \underline{w})$. The cost of opening a vacancy must be lower that the maximum possible profit.

$$\begin{cases} (S-1)\,(1-O(S,\theta))^S \left((Q-\underline{w}) \left(\frac{O(S,\theta)}{1-O(S,\theta)} \right) - \left(Q - \underline{w}\frac{S}{S-1} \right) (\ln(1-O(S,\theta))^{-1}) \right) = c, \\ \text{and} \\ (Q-\underline{w})\,(1-O(S,\theta))^{S-1} \left(1 - e^{-S/\theta} \right) = k. \end{cases}$$
$$(30)$$

The first equation determines the optimal number of applications as a function of c, $\underline{w}$, Q and θ. The second one determines the optimal value of θ as a function of k, $\underline{w}$, Q and S.

I represent these 2 equations in figure 10, for fixed values of $c = 0.001$, $k = 0.1$, $\underline{w} = 0$ and $Q = 1$.
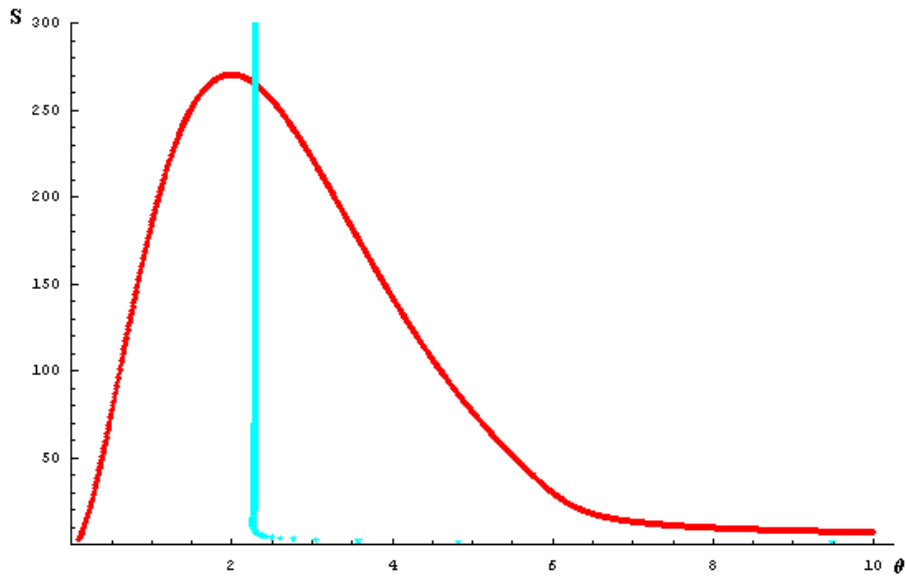

Figure 10: Equilibirum equations.

The equilibrium value of market tightness converges fast to a limiting value, that is independent of the number of applications. Using the limiting value of θ the second equation of the system is irrelevant and the first one describes properly the equilibrium as long as θ is correctly chosen. The reservation wage is relevant to compute θ, and this fact must be taken into account.

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4.1.2 Discussion about the free entry assumption

In any competitive market the free entry condition must hold. A firm that wants to enter the market creating a new job vacancy can do so. However, a difference between newly created firms and established firms may arise. A new firm must pay all the capital investment required to open a new vacancy, and also the cost of posting it. An established firm that loses a worker, needs only to afford the cost of posting the vacancy. The costs of opening and posting a vacancy are different, being the second one almost negligible with respect to the first one. An established firm will nearly always post a vacancy when it loses a worker, while new firms will arise only when the probability of getting a worker is high enough to incur in such investment. So, the number of vacancies in the market can be quite stable during long periods of time while the number of agents looking for a job can suffer great fluctuations in the short run. In such cases, a static analysis of the labor market using θ as an exogenous variable can be interesting.

4.2 Dynamic analysis. Steady State

The steady state equilibrium conditions of a dynamic set-up are not difficult to state. I solve the equilibrium under the free entry condition. This requires a discount rate for agents δ_a, for firms δ_f, and a job destruction rate $(1 - \gamma)$.

I solve first the firms side: A firm has an ex ante probability $\left(1 - e^{-S/\theta}\right)$ of receiving an application. The value of a vacancy Y can expressed as:

$$Y = -k + \delta_f \left(J(W)F(W) \left(1 - e^{-S/\theta}\right) + \left(1 - F(W) \left(1 - e^{-S/\theta}\right)\right) Y \right), \qquad (31)$$

where $J(W)$ is the value of a fulfilled vacancy at wage W.

As in the static case, firm's behavior offering wages is such that:

$$J(W)F(W) = J(\underline{w}) \left(1 - O\left(S, \theta\right)\right)^S, \qquad (32)$$

where

$$J(\underline{w}) = (Q - \underline{w}) + \delta_f \left(J(\underline{w})\gamma + Y(1 - \gamma) \right). \tag{33}$$

That is

$$J(\underline{w}) = \frac{(Q - \underline{w}) + \delta_f Y(1 - \gamma)}{(1 - \delta_f \gamma)}. \tag{34}$$

Then:

$$J(W)F(W) = \left(\frac{(Q - \underline{w}) + \delta_f Y(1 - \gamma)}{(1 - \delta_f \gamma)} \right) (1 - O(S, \theta))^S. \tag{35}$$

With this result and the free entry condition $(Y = 0)$ I get:

$$\frac{\delta_f}{1 - \delta_f \gamma} (Q - \underline{w}) (1 - O(S, \theta))^S \left(1 - e^{-S/\theta} \right) = k. \tag{36}$$

This is equal to the previous free entry result with the left-side multiplied by an exogenous constant $\dfrac{\delta_f}{1 - \delta_f \gamma}$. The profit associated to any wage offer is:

$$J(W) = \frac{\delta_f}{1 - \delta_f \gamma} (Q - W) \tag{37}$$

Then firms wage offer behavior is identical than in the static case since $\dfrac{\delta_f}{1 - \delta_f \gamma}$ cancels out. Then:

$$F(W) = \frac{Q - \underline{w}}{Q - W} (1 - O(S, \theta))^{S-1} \text{ for } W \in [\underline{w}, \overline{w}], \tag{38}$$

$$B(W) = \frac{(1 - O(S, \theta))}{O(S, \theta)} \left(\left(\frac{Q - \underline{w}}{Q - W} \right)^{\frac{1}{S-1}} - 1 \right). \tag{39}$$

The firm side remains identical to the one exposed in the static model.

Regarding agents there is now a substantial change. The reservation wage is endogenously determined by the value of unemployment. An agent rejects a wage offer if the value of being hired at that wage is not higher than the value of unemployment.

The unemployment value can be expressed as:

$$U = \delta_a \left(N(W) + (1 - O(S, \theta))^S U \right),$$
(40)

where $N(W)$ is the value of being hired at wage W. Since there is no wage offer made yet, $N(W)$ must be computed according to the expected wage in the labor market W_E. The value of being hired at the expected wage is given by:

$$N(W_E) = W_E + \delta_a \left(\gamma N(W_E) + (1 - \gamma) U \right).$$
(41)

Then

$$N(W_E) = \frac{W_E + \delta_a (1 - \gamma) U}{1 - \delta_a \gamma},$$

and

$$U = \frac{\delta_a W_E}{\left((1 - \delta_a \gamma) - \delta_a \left(\delta_a (1 - \gamma) + (1 - \delta_a \gamma) (1 - O(S, \theta))^S \right) \right)},$$
(42)

and W_E is obtained as the expected value for:

$$H(W) = (1 - O(S, \theta))^S \left(1 - \left(\frac{Q - \underline{w}}{Q - W} \right)^{\frac{S}{S-1}} \right) \text{ from } \underline{w} \text{ to } \overline{w},$$
(43)

where now $\underline{w} = U$ and $\overline{w} = Q - (Q - U)(1 - O(S, \theta))^{S-1}$

An equilibrium value of S must also be computed using the maximization of the expected return taking into account that now the reservation wage is determined by U. This is too complicated to solve but it will not change the main results obtained in the static model. U has, in equilibrium, a fixed value below Q. Setting the exogenous $\underline{w}$ to that value in the static model yields the same result as in the steady state equilibrium.

5 Extensions of the static model

I will present here a set of possible extensions of the static model that might be useful for other purposes.

5.1 Heterogeneous firms

In this scenario firms are heterogeneous. For simplicity I assume that there are only two type of firms. This implies two different labor markets. Agents can choose if they apply to one of them or in both. Moreover they can choose a different search effort in each market.

There are V_H high production firms that produce an amount Q_H if a vacancy is fulfilled and V_L low production firms that produce an amount $Q_L < Q_H$. The type of firm is observable. Agents make S_H applications to high production firms and S_L applications to low production firms. There are two values for market tightness θ_H and θ_L.

A complete characterization of the result remains for a future paper. As a sketch, the equilibrium result goes in the following direction. Being heterogeneous, firms will behave differently ex-ante. If agents are active in both markets, the domain of the wage offer distribution of different type of firms does not overlap, except in one of their end points in a way that the possible wage offer domain is connected. Low production firms offer wages from the reservation wage to a certain upper bound (lower than Q_L) and high production firms from that upper bound to a higher upper bound (lower than Q_H).

A low production firm then loses always against a high production firm offer and compete with the rest of low production firms. A high production firm competes only against other high production firms.

There is now an additional source of externality. Since S_L affects the value of the upper bound of the low wage offers, it changes also the lower bound of high wage offers. This effect is not taken into account by agents when they choose S_L.

5.2   Heterogeneous agents

Now agents are heterogenous. Again, for simplicity, there are just two types of workers, N_S skilled and N_U unskilled, and a single, ex-ante, homogeneous type of firm. Ex-post, a firm hiring a skilled worker produces Q_H and a firm hiring an unskilled agent produces $Q_L < Q_H$. Firms know if a worker is skilled or unskilled. They will offer wages according to different distributions depending on the type of the agent they choose. Both wage offer distributions will start from the reservation wage, having different upper bounds.

There are two possible scenarios. In the first one firms prefer skilled workers. They will make an offer to an unskilled worker only if there are no applications from skilled workers. This scenario is plausible but not interesting. In the second one the firm is, ex-ante, indifferent between making an offer to a skilled or to an unskilled worker. This happens when there is too much competition for skilled workers, driving down the expected profit of the firm trying to hire them. In this case firms first choose randomly to what type of worker they make an offer (if they have both type of applications), using a mixed strategy. Then they choose randomly one application from the selected group and make a wage offer extracted from the corresponding wage distribution. Since the wage distributions overlap, a skilled agent might end up with a lower wage than an unskilled agent.

5.3   Multiple wage offer rounds

In a more realistic set up firms can, after a rejection, choose a new application from the remaining set of applications and make a new offer. This implies a new round of wage offers. This additional round makes the model really complex. The ex-ante identical firms are now ex-post divided into three type of firms. Firms with no applications, that do not interact with the other two types. Firms with a single application, that can only make one offer in the first round of offers and do not participate in the second round. Firms with more than one application that can participate in both rounds. Again this is left for future research.

As a sketch, the equilibrium will go in the following direction. Firms with a single application can not participate in the second round if their offer is rejected. Those with more than one offer, in case of rejection, obtain the expected profit of the second round. This implies that the firms with one application will behave more aggressively in the first round and the firms with more than one application will behave less aggressively. In fact the behavior is nearly identical to the one observed between high production and low production firms. Firms with a single offer behave as high production firms. Willing to succeed in the first round they offer high wages. Firms with more applications behave as low production firms offering lower wages, since they have an outside option. At that point the difference with the heterogenous firm extension is that agents send their applications to an ex-ante homogenous set of firms. They do not know which firm will have a single application and which will have more than one. There is a unique labor market.

In the second round only those firms with more than one application compete. They do not know if the chosen application in the second round has already accepted an offer, neither the concrete amount of firms that will compete in the second round or the number of active applications that an agent has in the second round.

The result is again a piecewise defined return function for the agents.

6   Conclusions

The present model is a static, one-shot game, modelling a directed search labor market with an endogenously determined search effort. The wage bargaining is assumed to be a unique take-it-or-leave-it offer. Agents compete for a given number of job vacancies and firms compete to obtain a match in the market. This simple and intuitive model, explains most of the relevant features that might appear in a more complex model with endogenous market tightness or a dynamic set-up. Moreover, this type of static model is relevant when there are frictions in the demand side of the labor market and the number of posted vacancies does not react, or reacts slowly, to changes.

The result shows that a continuous wage distribution is obtained even under a complete, ex-ante, homogeneity. At the end agents will have different wages, even when they are identical and they did make the same search effort. This distribution reflects two thinks. The expected wage can be interpreted as an endogenously determined bargaining power. Agents gain power through their search effort and the realized wage, being stochastic, has a "luck" component. The particular wage that agents get is then a combination of effort and luck.

The reservation wage of agents becomes relevant per se. Any change in the reservation wage, such as rising unemployment benefits or minimum wage regulations, will produce effects in the whole distribution of wages, even when there is nobody whose real wage is the reservation wage. An increase in the reservation wage pushes up both the lower and the upper bound of the distribution changing its shape, affecting all the results: search effort, expected return, and unemployment.

Under the free-entry condition, an increase in the reservation wage reduces the return of agents, since market tightness decreases. If the number of vacancies is fixed, or firms can coordinate when posting vacancies, an increase in the reservation wage has positive effects on the return.

The relation between market tightness and search effort is not monotonic. It is possible to observe both procyclical or countercyclical responses of the search effort depending on the initial state of the labor market.

A negative externality arises due to the excessive competition between agents. There is an excessively high number of applications that reduces the expected return. There is not a clear way to avoid this externality, unless a coordination mechanism for agents can be implemented. This can be done through compulsory employment agencies, but his seams an unrealistic solution. An increase in the cost of applications has an ambiguous effect. It can have positive or negative effects depending on the exogenous parameters of the model.

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